FINAL REPORT TO

PACIFIC EARTHQUAKE ENGINEERING RESEARCH CENTER

LIFELINES PROGRAM TASK 1A01

TESTS OF 3D ELASTODYNAMIC CODES

Period of Performance: September 1, 1999 – September 30, 2000

Coordinating P. I.: Steven M. Day, San Diego State University
Co-Investigators: Jacobo Bielak, Carnegie-Mellon University
Doug Dreger & Shawn Larsen, University of California, Berkeley/ Lawrence Livermore National Laboratory
Robert Graves & Arben Pitarka, URS Corporation
Kim B. Olsen, University of California, Santa Barbara

September 10, 2001

INTRODUCTION

Numerical simulations of wave propagation can now be done in three dimensions for models with sufficient realism (e.g., three-dimensional geology, propagating sources, frequencies approaching 1 Hz) to be of engineering interest. However, before numerical simulations can be applied in the context of engineering studies or seismic hazard analysis, the numerical methods and the models associated with them must be thoroughly validated.

The Southern California Earthquake Center (SCEC) assembled a multi-institutional team from universities and the private sector to address this problem in collaboration with the PEER Lifelines Program. The current phase of this project, Task 1A01, focuses on validation of the underlying numerical methodologies and computer programs employed in numerical modeling of earthquake ground motion from propagating earthquakes in 3D earth models. The phase reported here was limited to idealized sources in simple earth structures. A subsequent phase, now in progress, is addressing more realistic earthquake sources and complex 3D earth structure.

Code verification is carried out through a systematic, coordinated program of test simulations. The project was designed to provide a foundation for further PEER/SCEC collaboration to model ground motion in urban sedimentary basins. By documenting the accuracy and limitations of existing numerical methods, the project has also produced a valuable stand-alone product for the ground motion community.

The coordinating PI for the project is Steven Day, of San Diego State University (SDSU). The participating modeling groups and key personnel are

Carnegie-Mellon University (CMU), Jacobo Bielak
Lawrence Livermore National Laboratory/University of California, Berkeley (UCB/ LLNL), Doug Dreger and Shawn Larsen
URS Corporation (URS), Robert Graves and Arben Pitarka
University of California, Santa Barbara, (UCSB), Kim Olsen

CODES

Five different codes were tested. These five codes are denoted by four-character abbreviations indicating the respective institutions: UCSB, UCBL, WCC1 (Robert Graves's URS code), WCC2 (Arben Pitarka's URS code), and CMUN. Of these, four are finite difference (FD), and one is finite element (FE).

All of the FD codes (**UCSB, UCBL**, **WCC1**, and **WCC2**) use uniform, structured grids, with staggered locations of the velocity and stress components and fourth-order accurate spatial differencing of the elastodynamic equations. The codes were independently programmed. The main variations among them include: degree of computational parallelism, type of memory management (e.g., main-memory contained operation versus roll-in/roll-out from disk), free-surface boundary condition formulation, absorbing boundary formulation, material interface representation (e.g., type of averaging of material properties in vicinity of properties gradients or interface), and source formulation.

The FE code (**CMUN**) uses unstructured gridding, with linear interpolation on tetralhedral elements. Grid generation is done serially (and is often the most time consuming part of a simulation), while equation solving is done in parallel execution, via an automated domain decomposition scheme.

OBJECTIVES

The project's overall objectives include the following:

1. Verify code components such as:
    - Internal equation solvers
    - Physical (free surface) boundary formulation
    - Artificial boundary formulation (absorbing boundaries)
    - Source formulation
    - Model building accuracy
    - . Anelastic loss formulation

2. Quantify and document code accuracy

3. Demonstrate code capability, with respect to
    - Source complexity
    - . Seismic velocity model complexity
    - Problem size
    - Anelastic attenuation

SCOPE OF PHASE I (TASK 1A01)

1. Test basic equation solvers, source formulation, and free surface boundary conditions, through comparisons with analytic solutions for uniform elastic halfspace problem.

2. Test accuracy and limitations of absorbing boundary conditions used to simulate radiation conditions at grid boundaries.

3. Test accuracy of material interface representation through comparisons with analytic solutions for layered halfspace problem.

4. Validate propagating earthquake source formulation.

The team designed and carried out four test simulations to meet these objectives. The test simulations were designed through a collaborative process, in which the coordinating PI modified his original problem proposal iteratively in response to co-PI feedback.

FORMAL PROBLEM DESCRIPTIONS

1. Problem UHS.1 (Figure 1)

Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

Material properties:

Uniform halfspace, Vp=6000 m/s, Vs=3464 m/s, density = 2700 kg/m^3, Qp=Qs=infinite

Source:

Point dislocation. The only non-zero moment tensor component is Mxy (equal to Myx), which has value $M_0=10^{18}$ Nm.

Moment-rate time history is $M_0*(t/T^2)*\exp(-t/T)$, where T=0.1 sec.

(Equivalently, the moment time history is $M_0*(1-(1+t/T)*\exp(-t/T))$, where T=0.1 sec).

Source Depth = 2000 m. That is, taking the epicenter as the origin, the source is at (0,0,2000).

Receivers:

Velocity time histories, in meters/sec, along free surface, at the 10 points

(600,800,0)
(1200,1600,0)
(1800,2400,0)
.
.
etc, up to (6000,8000,0).

That is, receivers are at 1000 m intervals along line oriented at angle 53.13 degrees (i.e., $\tan^{-1}(4/3)$) to the x axis.

Velocity components are to be given in the same coordinate system as above, i.e., v_x positive North, v_y positive East, and v_z positive down.

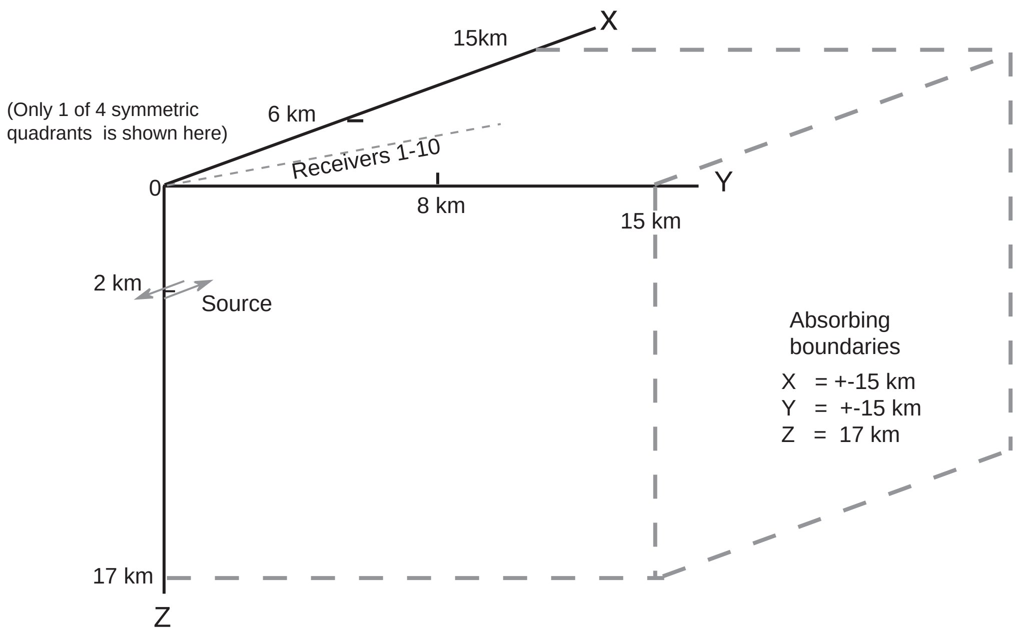

Fig. 1 Geometry for problem UHS.1

<u>Run time</u>

5 sec.

<u>Other Information</u>

Mesh size. Participants using uniform mesh 4/2 FD methods use a cell size of 100 m. for this test. Those using other methods should try to choose a cell size which will provide comparable accuracy.

Artificial boundaries. Place absorbing grid boundaries such that each boundary's orthogonal distance to the source point is 15,000 m. Distance of artificial boundaries of 15,000 m from source applies to all three directions, so, since the source is 2000 m deep, the bottom boundary should be at 17,000 m depth. In the case of distributed absorbers, distance refers to distance to the nearest point at which some significant artificial reflection may be generated.

<u>Output Instructions</u>

Solutions are to be compared with each other and with independent solutions over the bandwidth 0 to 5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function.

2. Problem UHS.2 (Figure 2)

Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

Material properties:

Uniform halfspace, Vp=6000 m/s, Vs=3464 m/s, density = 2700 kg/m^3, Qp=Qs=infinite

Source:

Point dislocation. The only non-zero moment tensor component is Mxy (equal to Myx), which has value $M_0=10^{18}$ Nm.

Moment-rate time history is $M_0*(t/T^2)*\exp(-t/T)$, where T=0.1 sec.

(Equivalently, the moment time history is $M_0*(1-(1+t/T)*\exp(-t/T))$, where T=0.1 sec).

Source Depth = 2000 m. That is, taking the epicenter as the origin, the source is at (0,0,2000).

Receivers:

Velocity time histories, in meters/sec, along free surface, at the 10 points

(600,800,0)
(1200,1600,0)
(1800,2400,0)
.
.
etc, up to (6000,8000,0).

That is, receivers are at 1000 m intervals along line oriented at angle 53.13 degrees (i.e., $\tan^{-1}(4/3)$) to the x axis.

Velocity components are to be given in the same coordinate system as above, i.e., v_x positive North, v_y positive East, and v_z positive down.

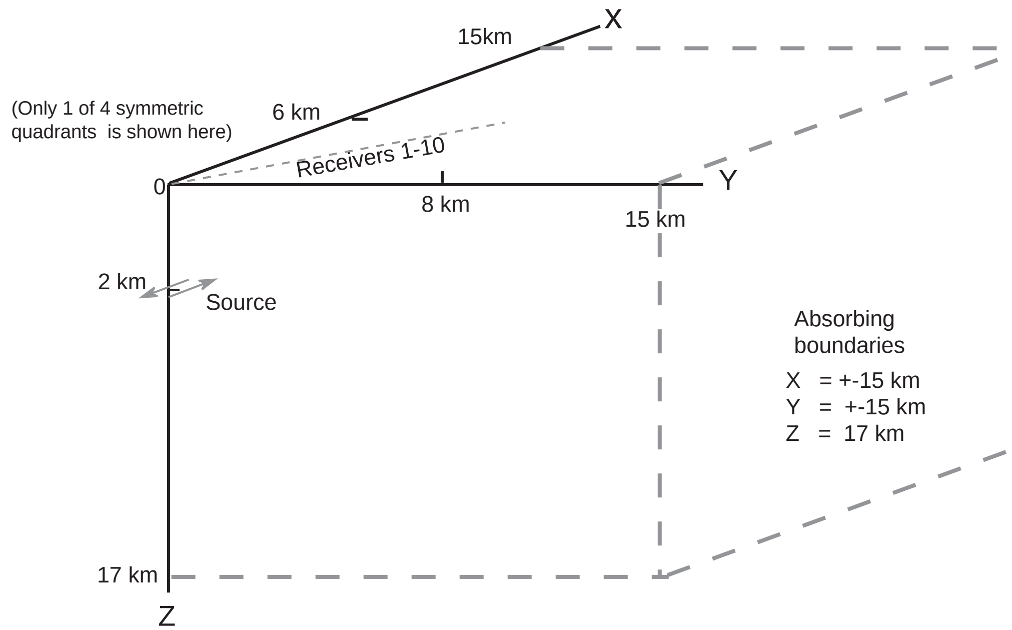

Fig. 1 Geometry for problem UHS.1

Run time

5 sec.

Other Information

Mesh size. Participants using uniform mesh 4/2 FD methods use a cell size of 100 m. for this test. Those using other methods should try to choose a cell size which will provide comparable accuracy.

Artificial boundaries. Place absorbing grid boundaries such that side boundaries' orthogonal distances to the source point are 10,000 m, and bottom boundary is at depth of 10,000 m. Some codes use an artificially attenuative zone ("sponge zone") between the main part of the grid and the absorbing boundaries. In that case, the location of the absorbing boundary will be taken to be the beginning of the sponge zone. To ensure meaningful comparisons among codes, the size of any sponge zone must not add more than 20% to the grid in any of the 3 lattice directions.

Output Instructions

Solutions are to be compared with each other and with independent solutions over the bandwidth 0 to 5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function.

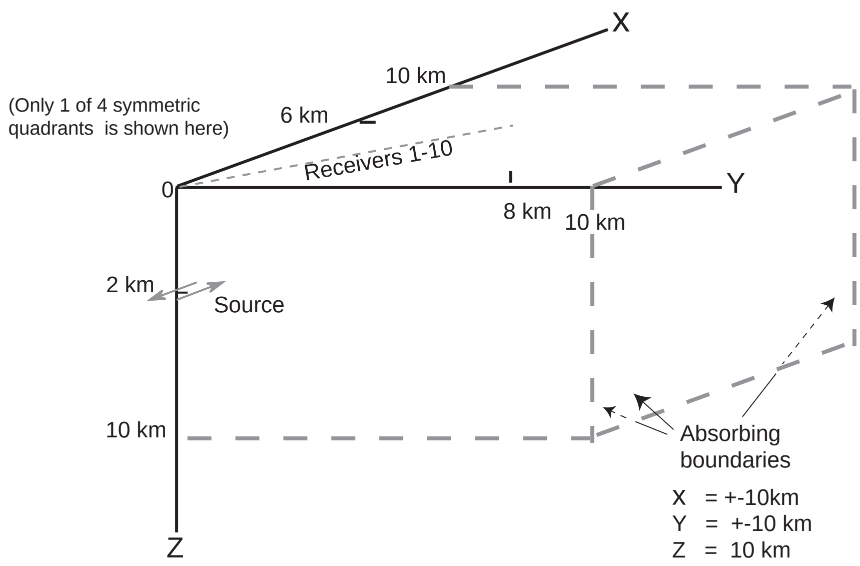

Fig. 2 Geometry for problem UHS.2

3. Problem LOH.1 (Figure 3)

<u>Coordinate System</u>

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

<u>Material Properties</u>

The top 1000 m has Vs=2,000 m/s, Vp=4,000 m/s, density=2600 kg/m^3.

The underlying halfspace has (as before) Vs=3,464 m/s, Vp=6,000 m/s, density=2700 kg/m^3

Both Q's are infinite everywhere.

<u>Source:</u>

Point dislocation. The only non-zero moment tensor component is Mxy (equal to Myx), which has value $M_0=10^{18}$ Nm.

Moment-rate time history is $M_0*(t/T^2)*\exp(-t/T)$, where T=0.1 sec.

(Equivalently, the moment time history is $M_0*(1-(1+t/T)*\exp(-t/T))$, where T=0.1 sec).

Source Depth = 2000 m. That is, taking the epicenter as the origin, the source is at (0,0,2000).

Receivers:

Velocity time histories, in meters/sec, along free surface, at the 10 points

(600,800,0)
(1200,1600,0)
(1800,2400,0)
.
.
etc, up to (6000,8000,0).

That is, receivers are at 1000 m intervals along line oriented at angle 53.13 degrees (i.e., $\tan^{-1}(4/3)$) to the x axis.

Velocity components are to be given in the same coordinate system as above, i.e., v_x positive North, v_y positive East, and v_z positive down.

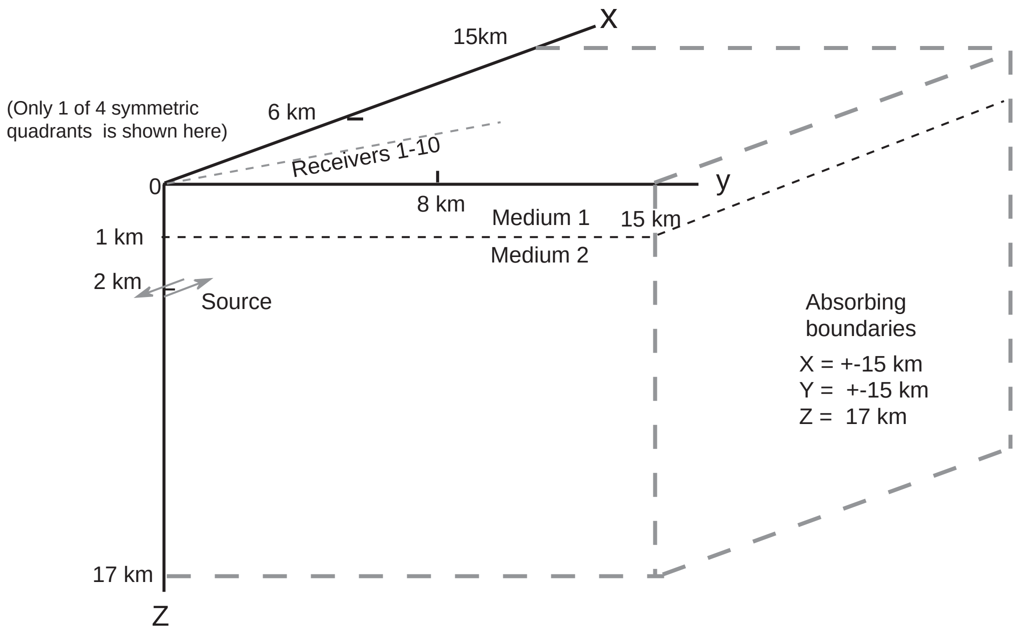

Fig. 3 Geometry for problem LOH.1

Run time

9 sec.

Other Information

Mesh size. Participants using uniform mesh 4/2 FD methods use a cell size of 100 m. for this test. Those using other methods should try to choose a cell size which will provide comparable accuracy.

Artificial boundaries. Place absorbing grid boundaries such that each boundary's orthogonal distance to the source point is 15,000 m. Distance of artificial boundaries of 15,000 m from source applies to all three directions, so, since the source is 2000 m deep, the bottom boundary should be at 17,000 m depth. In the case of distributed absorbers, distance refers to distance to the nearest point at which some significant artificial reflection may be generated.

Output Instructions

Solutions are to be compared with each other and with independent solutions over the bandwidth 0 to 5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function.

4. Problem LOH.2 (Figure 4)

Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

Material Properties

The top 1000 m has Vs=2,000 m/s, Vp=4,000 m/s, density=2600 kg/m^3.

The underlying halfspace has (as before) Vs=3,464 m/s, Vp=6,000 m/s, density=2700 kg/m^3

Both Q’s are infinite everywhere.

Source

Finite Fault in the vertical plane $x = 0$ (see Fig 4a).
Right-lateral strike-slip.
Region of slip delimited by rectangle with following vertices:
(0, 0, 2000); (0, 8000, 2000); (0, 0, 6000); (0, 8000, 6000)
Hypocenter at $(x_H, y_H, z_H) = (0, 1000, 4000)$

Fault surface coordinates (ξ, η) defined by (see Fig 4b)

$\xi = y$

$\eta = z - 2000$

In the fault coordinate system, the hypcenter is denoted by (ξ_H, η_H), and

$$(\xi_H, \eta_H) = (1000, 2000)$$

In terms of fault-plane basis vectors $\hat{\xi}, \hat{\eta}$, the slip vector is $\hat{\xi} S(\xi, \eta, t)$, where the slip function S has same shape and amplitude everywhere within the fault surface, but is time-shifted by an amount proportional to the distance of (ξ, η) from the hypocenter (i.e., the slip front propagates at a constant rupture velocity). S is given by

$$S(\xi, \eta, t) = S_0 \left[1 - (1 + \tau / T) e^{-\tau/T} \right] H(\tau)$$

where H is the Heaviside step function, the time relative to rupture arrival, τ, is

$$\tau = t - V_{rup}^{-1} \left[(\xi - \xi_H)^2 + (\eta - \eta_H)^2 \right]^{1/2},$$

the static slip S_0 is 1 meter, the rupture velocity V_{rup} is 3000 m/sec, and the smoothing time, T, is 0.1 sec.

Equivalently, the slip velocity function is

$$\dot{S}(\xi, \eta, t) = S_0 \left(\tau / T^2 \right) e^{-\tau/T} H(\tau)$$

Receivers:

Velocity time histories, in meters/sec, along free surface, at the 10 points

(600,800,0)
(1200,1600,0)
(1800,2400,0)
.
.
etc, up to (6000,8000,0).

That is, receivers are at 1000 m intervals along line oriented at angle 53.13 degrees (i.e., $\tan^{-1}(4/3)$) to the x axis.

Velocity components are to be given in the same coordinate system as above, i.e., v_x positive North, v_y positive East, and v_z positive down.

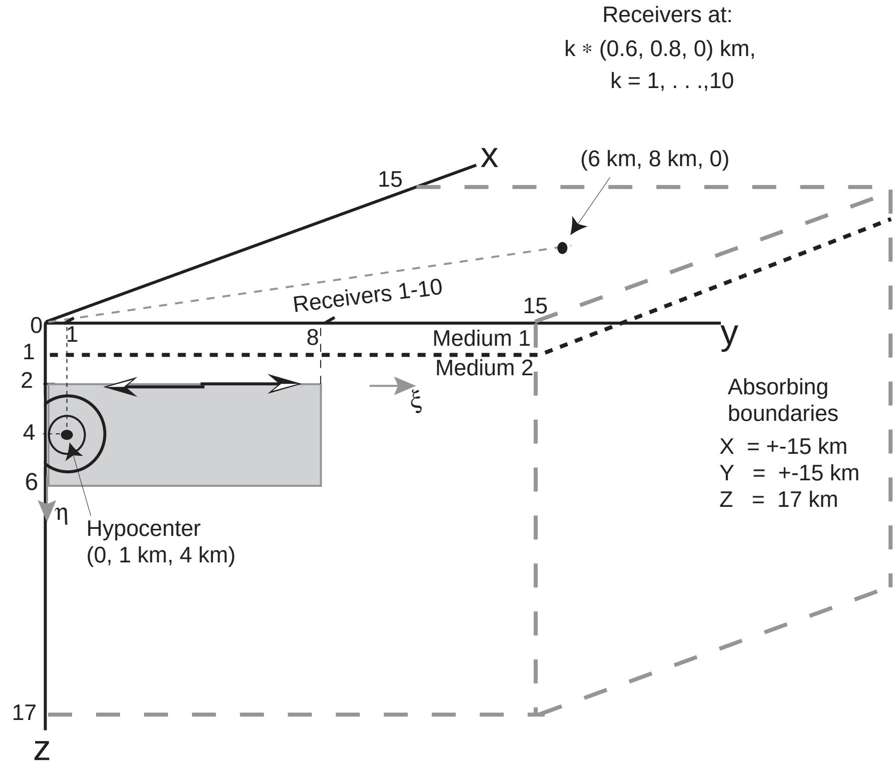

Fig. 4a Geometry for problem LOH.2

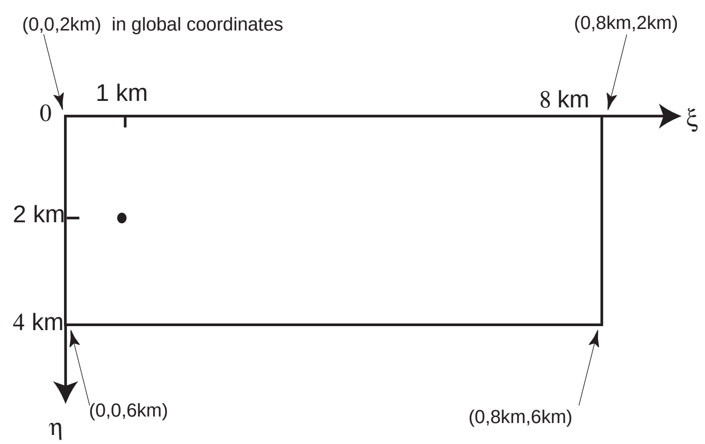

Fig 4b. Fault-surface coordinates for problem LOH.2

<u>Run time</u>

9 sec.

Other Information

Mesh size. Participants using uniform mesh 4/2 FD methods use a cell size of 100 m. for this test. Those using other methods should try to choose a cell size which will provide comparable accuracy.

Artificial boundaries. Place absorbing grid boundaries such that each boundary's orthogonal distance to the source point is 15,000 m. Distance of artificial boundaries of 15,000 m from source applies to all three directions, so, since the source is 2000 m deep, the bottom boundary should be at 17,000 m depth. In the case of distributed absorbers, distance refers to distance to the nearest point at which some significant artificial reflection may be generated.

Output Instructions

Solutions are to be compared with each other and with independent solutions over the bandwidth 0 to 5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function.

OUTPUT FORMAT

Each problem generates one ascii output file. The file name consists of the 4-character abbreviation denoting the code used, followed by a dot and the problem name, e.g., "wcc2.uhs.1".

The file format should be as illustrated by the write statements in the model Fortran code shown below. The Fortran model is followed by a Matlab m-file (macro) which reads the file produced by the Fortran program, and which can be used to test the file formating. The file name 'PGETEST.1' in the line that reads fid=fopen('PGETEST.1') should be changed to the name of the file being read.

The output file will contain scalars nr, nt, and dt, which are the number of receiver points, number of time points in each time series, and time step size, respectively.

The output file also contains 2 1D arrays giving the output coordinate points. x(i) and y(i), i=1:nr, contain the x and y coordinates of the receivers. For UHS.1, example, x(1)=600, x(2)=1200, . . ., x(10)=6000.

The rest of the output file consists of several 2D arrays giving the time series.

Array t contains the solution times, t(j,i) being the time associated with the solution at receiver i at time-point j (i.e., the nt*nr time values are written with the time index most rapidly varying). In the usual case of a constant time step, t(j,i) is just j*dt, for all i.

Array vx contains x-component (positive North) velocities, vx(j,i) being the x velocity at time j at receiver point i (i.e., the time index is most rapidly varying).

Arrays vy and vz are the same as vx, but containing the y and z components of velocity, respectively.

Fortran Illustration of Output

```
c*****************************************************************
dimension t(2000,100), vx(2000,100), vy(2000,100), vz(2000,100)
dimension x(100),y(100)
dt=.02
dx=600.
dy=800.
pi=4.*atan(1.)
period=1.0
nr=10
nt=1000
do 1 i=1,nr
x(i)=float(i)*dx
y(i)=float(i)*dy
amp=float(i)
do 1 j=1,nt
t(j,i)=float(j)*dt
vx(j,i)=amp*sin(pi*t(j,i)/period)
vy(j,i)=amp*cos(pi*t(j,i)/period)
vz(j,i)=amp*t(j,i)*exp(-t(j,i)/period)
1 continue
open (10,file='PGETEST.1')
rewind (10)
write(10,*) nr,nt,dt
write(10,*)  (x(i),i=1,nr)
write(10,*)  (y(i),i=1,nr)
write(10,*) ((t(j,i),j=1,nt),i=1,nr)
write(10,*) ((vx(j,i),j=1,nt),i=1,nr)
write(10,*) ((vy(j,i),j=1,nt),i=1,nr)
write(10,*) ((vz(j,i),j=1,nt),i=1,nr)
end
```

Matlab macro to read file

```
% MATLAB macro
%
% Reads file PGETEST.1 generated by WriteFortran.f
%
% Test of ASCII format for PG&E/SCEC code verification
```

```
%
% Reads output generated by fortran program WriteFortran,
% which sits in /usr12/day/PGE/TestFormat
%
% S. Day   6jan99
%
fid=fopen('PGETEST.1')
A=fscanf(fid,'%g',[3]);
nr=A(1);nt=A(2);dt=A(3);
x=fscanf(fid,'%g',[nr]);
y=fscanf(fid,'%g',[nr]);
t=fscanf(fid,'%g',[nt,nr]);
vx=fscanf(fid,'%g',[nt,nr]);
vy=fscanf(fid,'%g',[nt,nr]);
vz=fscanf(fid,'%g',[nt,nr]);
fclose(fid);
for k=1:nr
figure(k)
plot(t(:,k),vx(:,k),'r',t(:,k),vy(:,k),'b',t(:,k),vz(:,k),'g')
end
```

TEST RESULTS

Status of Codes at Start of Project

At the initiation of the project, we invited the project participants, as well as other SCEC scientists working in 3D wave propagation, to participate in a preliminary code testing exercise. In addition to codes CMUN, UCBL, WCC1, WCC2, and UCSB, a FD code from another university group was represented at this stage. Figure 5 shows a sample of the results for Problem UHS.1, for the radial component of velocity at the receiver at 10 km epicentral distance from the source (original source deconvolved and replaced with a Gaussian of spread 0.05 s). A few of the solutions are in close agreement with each other and with a reference solution computed by the reflectivity method (an integral transform technique, requiring numerical quadrature). There in enormous variability among the others. Thus, some participants had codes that accurately reproduced the reference solution for the uniform halfspace, while others had significant inaccuracies at this stage.

Problem UHS.1

Figure 6 shows the results for UHS.1 (again with Gaussian source of spread 0.05 s) after a cycle of testing and feedback. Now all methods are in close agreement with each other. In fact, the semi-analytic reference solution differs more from the numerical solutions than the latter do with each other. It has since been verified that the numerical solutions are more accurate than the semi-analytic solution in this case, due to small integration errors in the latter which have been corrected in subsequent work (Figs 7 and 8).

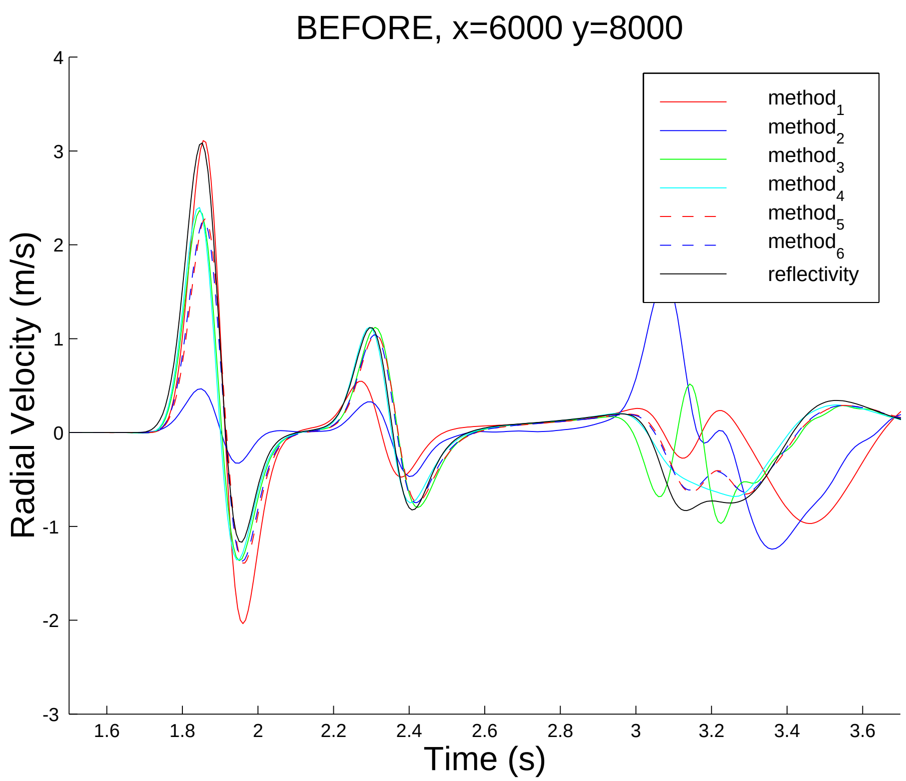

Figure 5. Initial numerical solutions for Problem UHS.1.

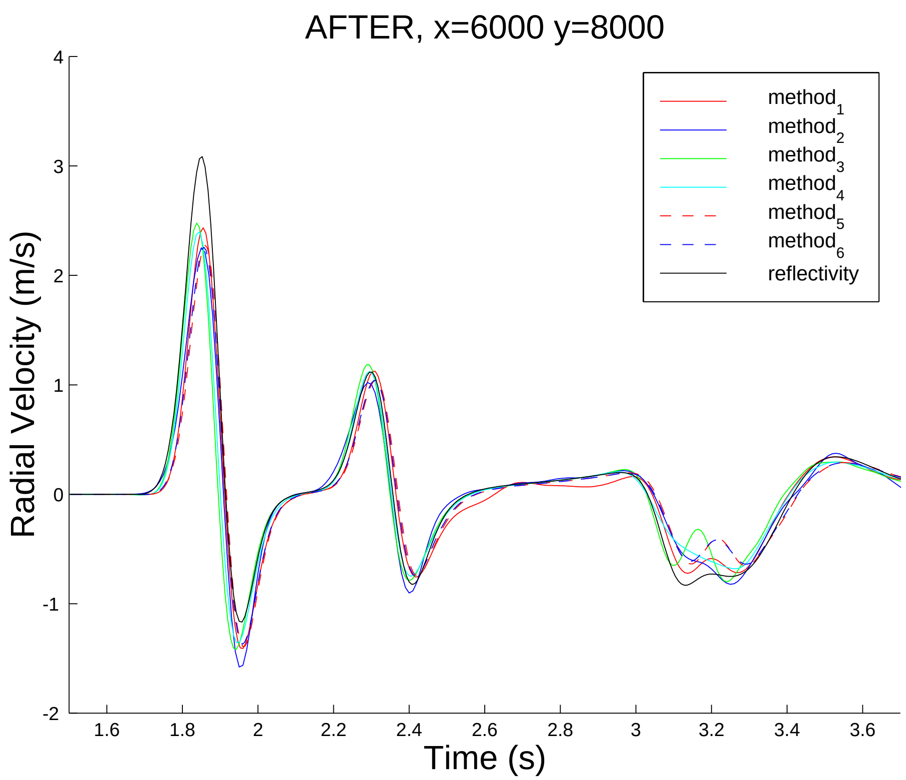

Figure 6. Numerical solutions for Problem UHS.1 after the testing cycle .

Problem UHS.2

Problem UHS.2

Figure 7 shows the results for the uniform halfspace test with absorbing boundaries. Source has again been deconvolved and replaced with Gaussian, spread 0.05 s.

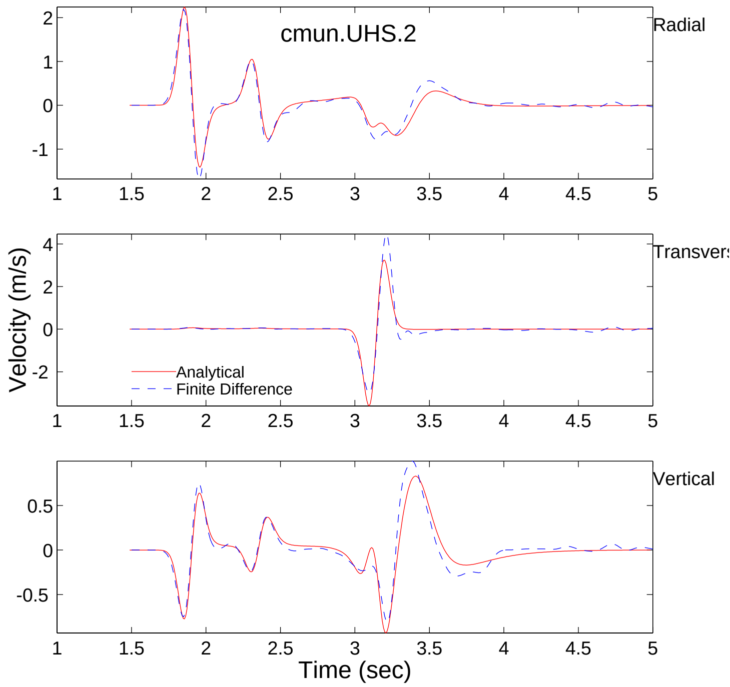

Fig. 7a. Comparison of numerical (CMUN) and analytical solutions for UHS.2.

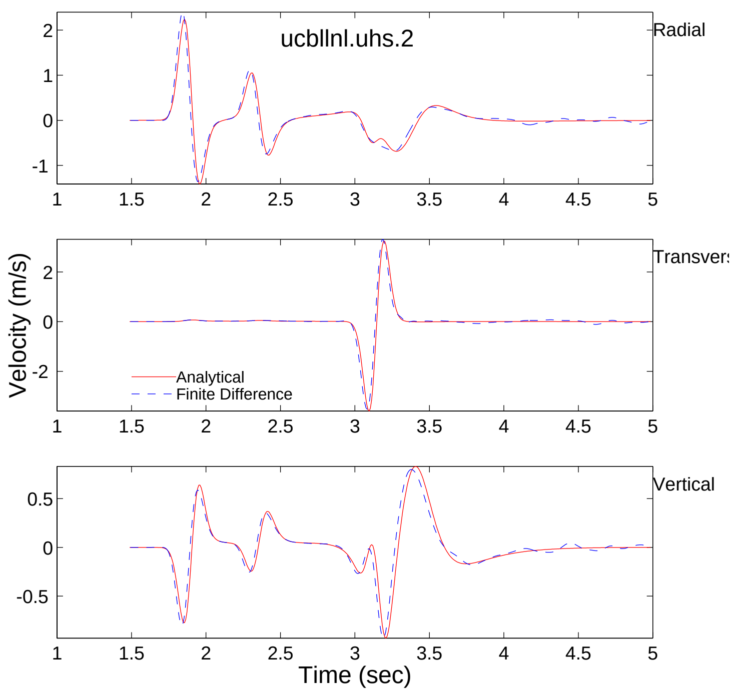

Fig. 7b. Comparison of numerical (UCBL) and analytical solutions for UHS.2.

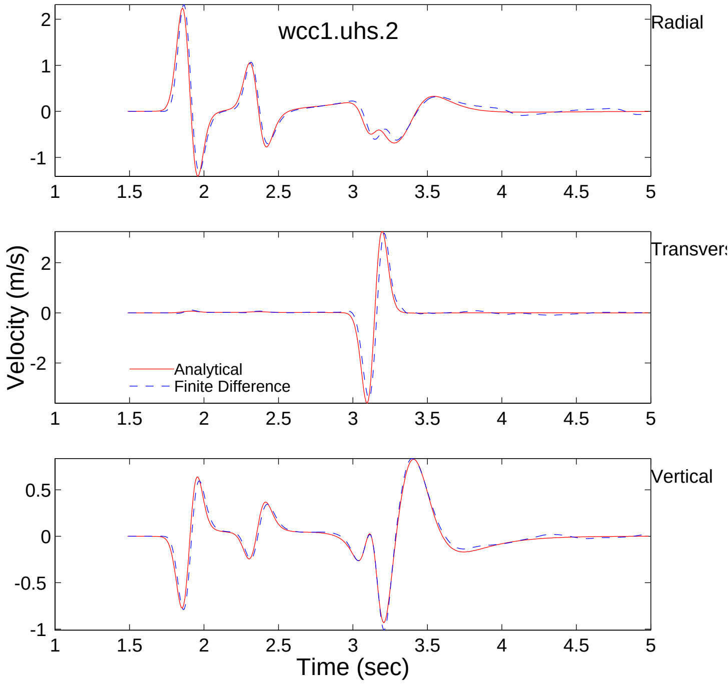

Fig. 7c. Comparison of numerical (WCC1) and analytical solutions for UHS.2.

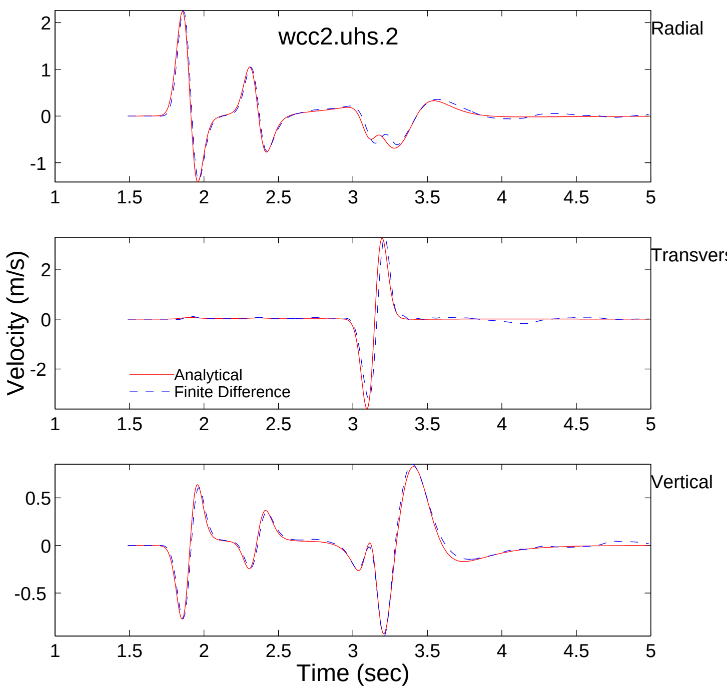

Fig. 7d. Comparison of numerical (WCC2) and analytical solutions for UHS.2.

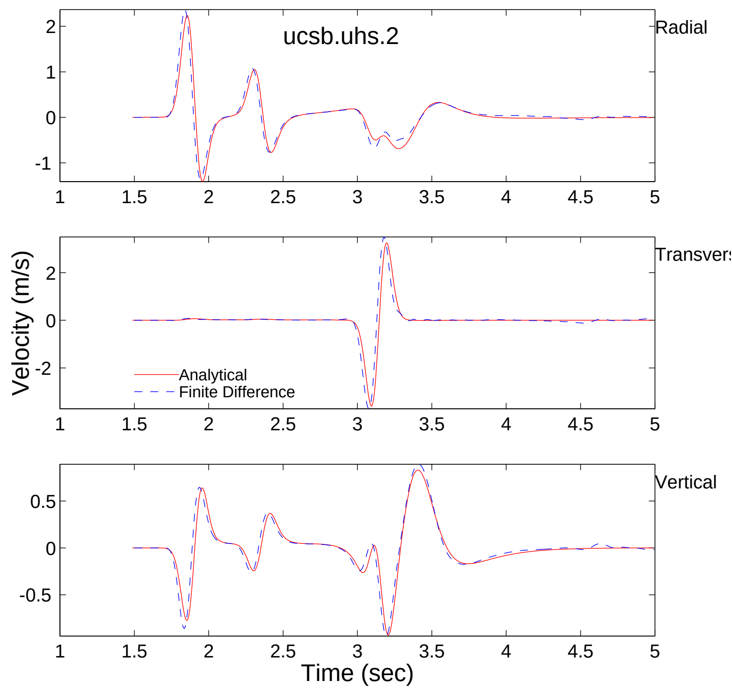

Fig. 7e. Comparison of numerical (UCSB) and analytical solutions for UHS.2.

Problem LOH.1

Figure 8 shows the results for the layer over halfspace test, with point dislocation source (Gaussian time function, spread 0.06 s).

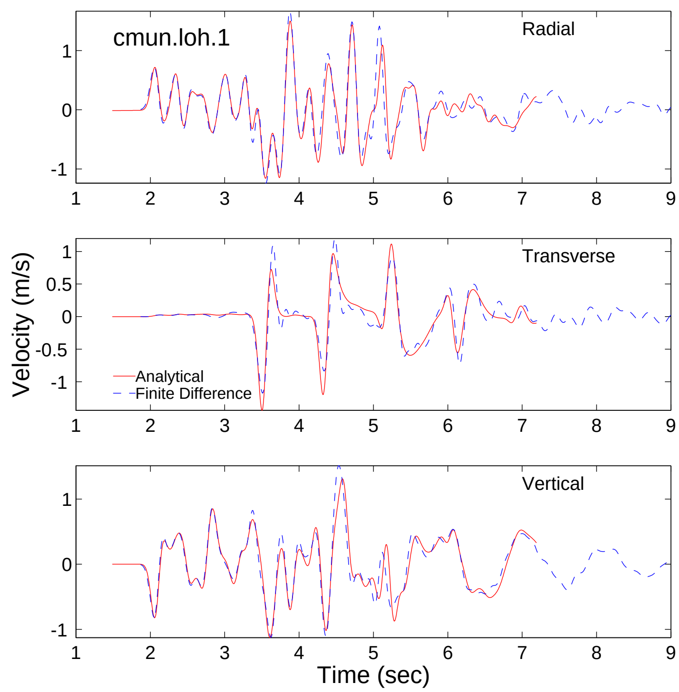

Fig. 8a. Comparison of numerical (CMUN) and analytical solutions for LOH.1.

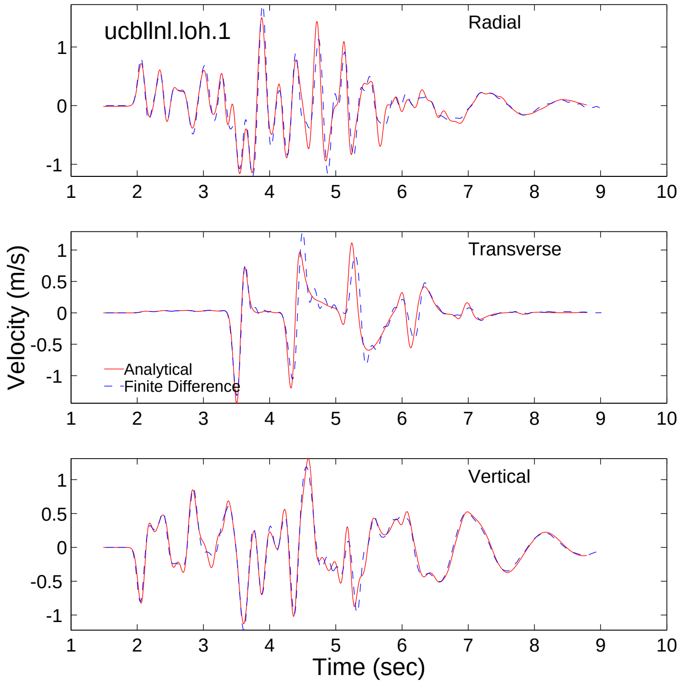

Fig. 8b. Comparison of numerical (UCBL) and analytical solutions for LOH.1.

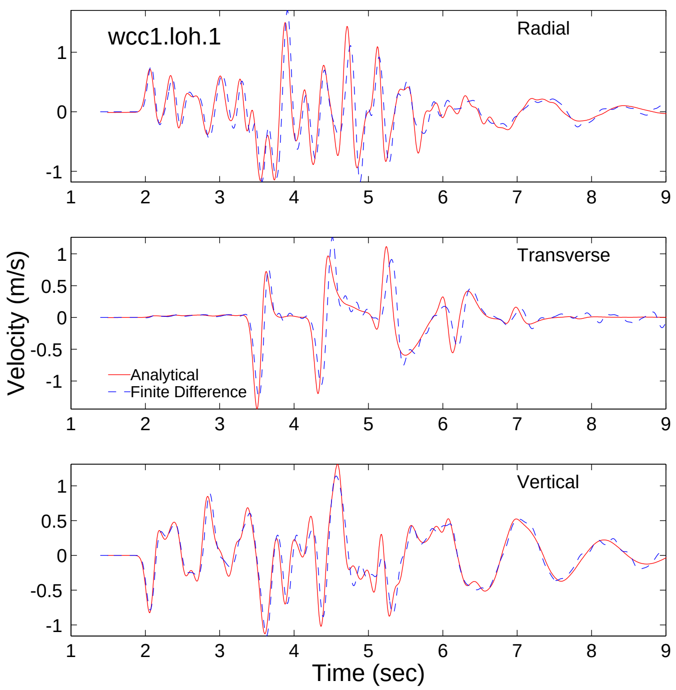

Fig. 8c. Comparison of numerical (WCC!) and analytical solutions for LOH.1.

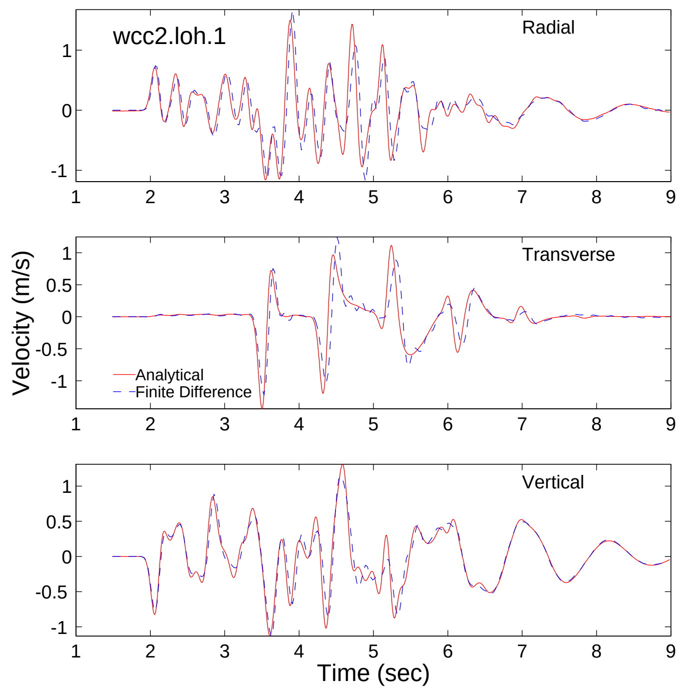

Fig. 8d. Comparison of numerical (WCC2) and analytical solutions for LOH.1.

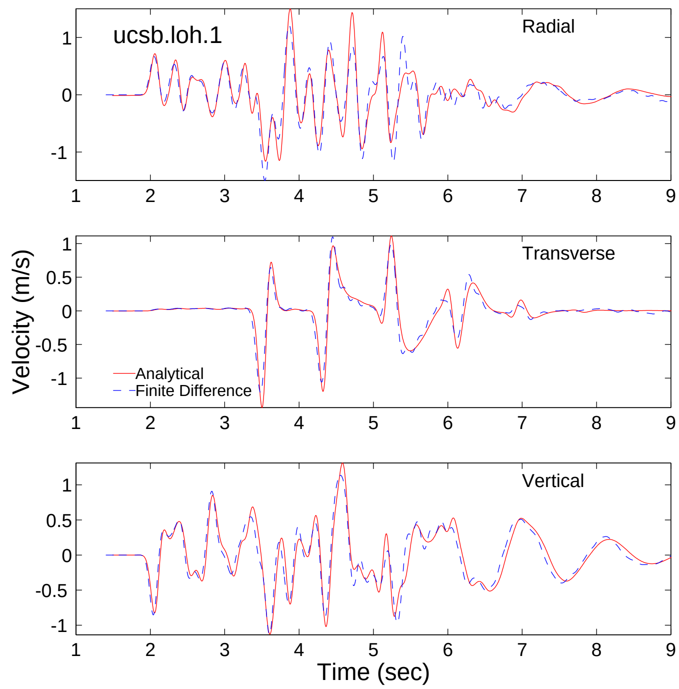

Fig. 8e. Comparison of numerical (UCSB) and analytical solutions for LOH.1.

Problem LOH.2

Figure 9 shows the results for the layer over halfspace test, with propagating dislocation source (Gaussian time function of slip, spread 0.06 s). In this case, we have used the UCBL solution as a common reference (red curve) against which the others are plotted for comparison.

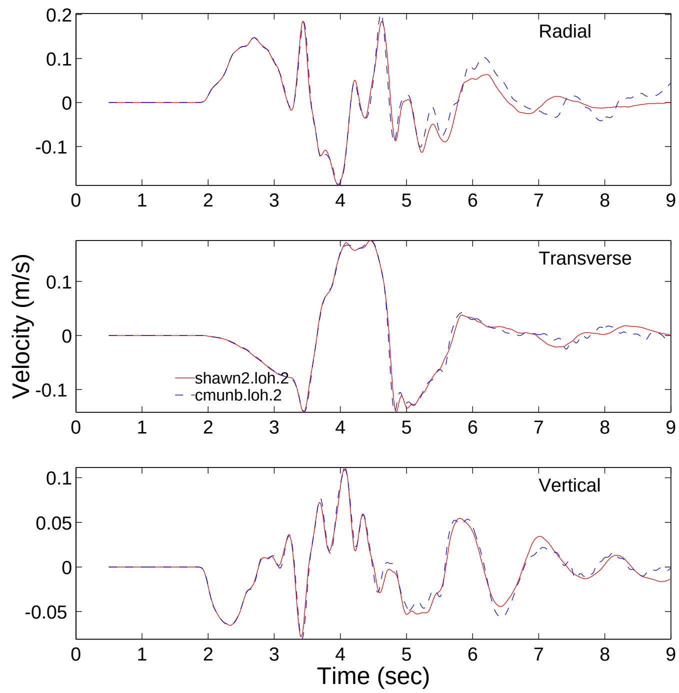

Fig. 9a. Comparison of one numerical solution (CMUN) with another (UCBL) for LOH.2.

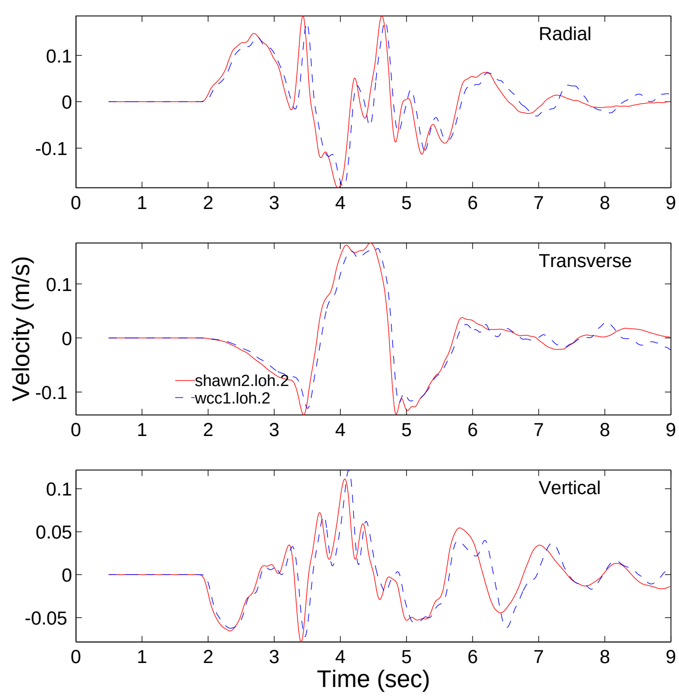

Fig. 9b. Comparison of one numerical solution (WCC1) with another (UCBL) for LOH.2.

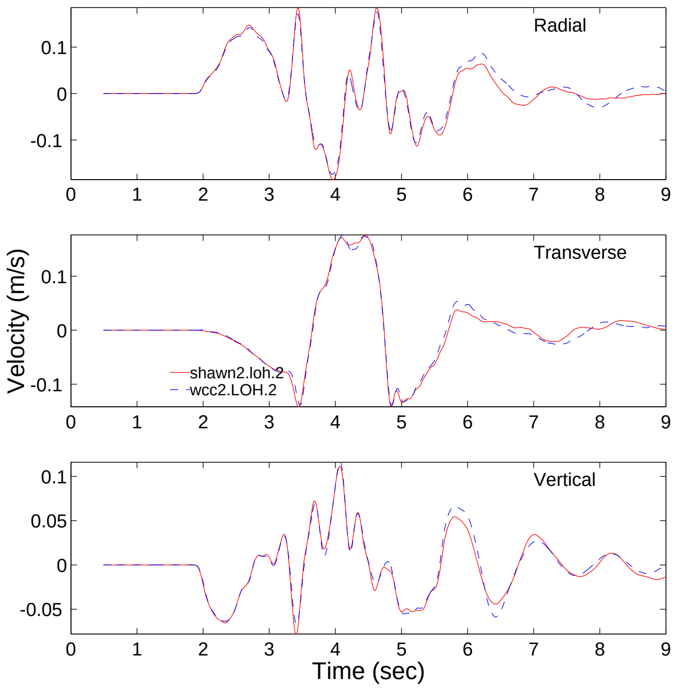

Fig. 9c. Comparison of one numerical solution (WCC2) with another (UCBL) for LOH.2.

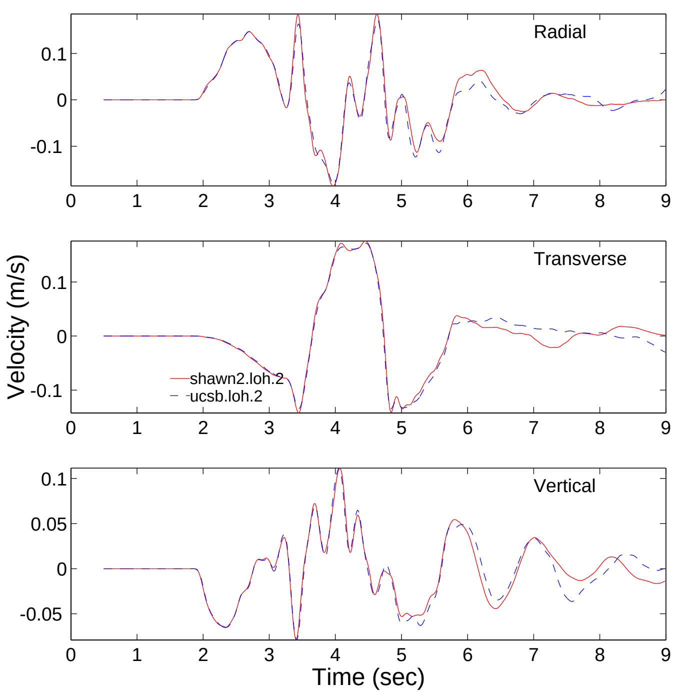

Fig. 9d. Comparison of one numerical solution (UCSB) with another (UCBL) for LOH.2.

SUMMARY

All 4 of the test problems for this first phase of the study have been completed by all four participating groups, and with all five codes. Agreement among the codes appears to be very good, as judged by a waveform comparison carried out over a bandwidth which covers the full wavelength range over which accuracy is to be expected of the underlying FD and FE methods. These comparisons set the stage for a subsequent phase in which complex seismic velocity structure, anelastic attenuaton, and realistic sources will be modeled.