

FINAL REPORT TO  
PACIFIC EARTHQUAKE ENGINEERING RESEARCH CENTER

LIFELINES PROGRAM TASK 1A02

**TESTS OF 3D ELASTODYNAMIC CODES**

Period of Performance: October 1, 2000 – March 31, 2002

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October 10, 2003

## INTRODUCTION

Numerical simulations of wave propagation can now be done in three dimensions for models with sufficient realism (e.g., three-dimensional geology, propagating sources, frequencies approaching 1 Hz) to be of engineering interest. However, before numerical simulations can be applied in the context of engineering studies or seismic hazard analysis, the numerical methods and the models associated with them must be thoroughly validated.

The current report describes progress made under Task 1A02, which aims to validate numerical modeling of earthquake ground motion from propagating earthquakes in 3D earth models. An earlier phase (Task 1A01) was limited to idealized sources in simple earth structures. The current phase addresses more realistic earthquake sources and complex 3D earth structure. The project was designed to provide a foundation for further PEER/SCEC collaboration to model ground motion in urban sedimentary basins.

The coordinating PI for the project is Steven Day, of San Diego State University (SDSU). The participating modeling groups and key personnel are

Carnegie-Mellon University (CMU), Jacobo Bielak  
Lawrence Livermore National Laboratory/University of California, Berkeley  
(UCB/ LLNL), Doug Dreger and Shawn Larsen  
URS Corporation (URS), Robert Graves and Arben Pitarka  
University of California, Santa Barbara, (UCSB), Kim Olsen

## CODES

Five different codes were tested. These five codes are denoted by four-character abbreviations indicating the respective institutions: UCSB, UCBL, WCC1 (Robert Graves's URS code), WCC2 (Arben Pitarka's URS code), and CMUN. Of these, four are finite difference (FD), and one is finite element (FE).

All of the FD codes (**UCSB**, **UCBL**, **WCC1**, and **WCC2**) use uniform, structured grids, with staggered locations of the velocity and stress components and fourth-order accurate spatial differencing of the elastodynamic equations. The codes were independently programmed. The main variations among them include: degree of computational parallelism, type of memory management (e.g., main-memory contained operation versus roll-in/roll-out from disk), free-surface boundary condition formulation, absorbing boundary formulation, material interface representation (e.g., type of averaging of material properties in vicinity of properties gradients or interface), and source formulation.

The FE code (**CMUN**) uses unstructured gridding, with linear interpolation on tetrahedral elements. Grid generation is done serially (and is often the most time consuming part of a simulation), while equation solving is done in parallel execution, via an automated domain decomposition scheme.

## SCOPE OF TASK 1A02

1. Test accuracy of codes in presence of complex earth structure, as represented by the SCEC Reference 3D Seismic Velocity Model
2. Test accuracy and limitations of anelastic attenuation models
3. Test accuracy of propagating thrust fault source representation
4. Perform simulations of Northridge earthquake.

## FORMAL PROBLEM DESCRIPTIONS

### 1. Problem SC2.1

#### Overview

This problem is for a complex 3D model, intended to be a realistic seismic velocity model for the San Fernando Valley/Los Angeles Basin region of southern California. The source-receiver geometry is sketched in Figure 1.

#### Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

#### Mapping to Geographical Coordinates

In order to map the southern California velocity structure (which is specified in lat, lon.) onto this coordinate system, map latitude onto meters north, and map longitude onto meters east, according to the following recipe:

SW corner is at (33.7275238 latitude, -118.90798187 long). This corner maps to origin, i.e.,  $(x,y) = (0\text{m}, 0\text{ m})$ .

NW corner is at (34.44875336, -118.90798187), and maps to  $(x,y) = (80000\text{ m}, 0\text{ m})$ .

SE corner, (33.7275238, -118.04201508), maps to  $(x,y) = (0\text{ m}, 80000\text{ m})$ .

NE corner, (34.44875336, -118.04201508), maps to  $(x,y) = (80000\text{ m}, 80000\text{ m})$ .

This mapping neglects earth curvature, and should be equivalent to  $1^\circ\text{ lat} = 110.922\text{ km}$ ,  $1^\circ\text{ lon} = 92.382\text{ km}$ .

Note: The above corners were specified because they were convenient for defining the coordinate mapping. Actual grid may be cropped within, or extended beyond, this 80 x 80 x 30 km region.

### Material Properties.

This problem uses the SCEC Southern California Seismic Velocity Model, Version 2, except for modifications described below to impose a lower limit on the velocities. The unmodified model is described in the following publication:

Magistrale, H., S. M. Day, R. Clayton, and R.W. Graves (2000). The SCEC southern California reference three-dimensional seismic velocity model version 2, *Bull. Seism. Soc. Am.*, 90, S65-S76.

The model is available as a Fortran code by anonymous ftp:

`ftp://moho.sdsu.edu/pub/Version2/Version2.2.tar.Z`

and its use is described in the file “read\_me” in the same directory. Questions concerning the SCEC velocity model should be directed to Harold Magistrale ([harold@moho.sdsu.edu](mailto:harold@moho.sdsu.edu)). Given a list of points, by lat (degrees), long (degrees), depth (meters), the code returns  $V_p$ ,  $V_s$  (both in m/s), and density ( $\text{kg/m}^3$ ). Details and examples are in the read\_me file.

The SCEC model is to be modified as follows for this computation: Replace the SCEC model S velocity with the value 200 m/s whenever the SCEC model value falls below 200 m/s. Whenever this minimum S velocity is imposed, the P wave velocity is set equal to 3 times the S velocity (600 m/s in this case). Density values follow the SCEC model without modification.

For this test,  $Q_p$  and  $Q_s$  will both be infinite.

### Source

[Source is similar to that of the previous point source problems, but with a different depth and different time constant (T), corresponding to the increased spatial scale (and lower maximum frequency) of the calculation.]

Point dislocation. The only non-zero moment tensor component is  $M_{xy}$  (equal to  $M_{yx}$ ), which has value  $M_0=10^{18}$  Nm.

Moment-rate time history is  $M_0*(t/T^2)*\exp(-t/T)$ , where  $T=0.2$  sec.

(Equivalently, the moment time history is  $M_0*(1-(1+t/T)*\exp(-t/T))$ , where  $T=0.2$  sec).

Hypocentral coordinates (56000 m, 40000 m, 14000 m)

(As a check, this should put the epicenter slightly east of Northridge.)

### Receivers

Velocity time histories, in meters/sec, along free surface, at 8 points. The 8 points are along a line through the epicenter, oriented at angle  $-53.13$  (i.e.,  $-\tan^{-1}(4/3)$ ) to the x axis. The receiver coordinates are expressed here in km to make them easier to read (but all output files are still to be in mks units):

$$(80 - j*6, 8 + j*8), j = 1, \dots, 8$$

That is, the points

(74km, 16km)  
(68km, 24km)  
(62km, 32km)  
(56km, 40km)  
(50km, 48km)  
(44km, 56km)  
(38km, 64km)  
(32km, 72km)

Velocity components are to be given in the same coordinate system as above, i.e.,  $v_x$  positive North,  $v_y$  positive East, and  $v_z$  positive down.

Output format is as given in the Output Format section below.

Output file suffix (see Output Format section) is SC2.1 (to indicate Southern California model, Version 2, problem number 1).

### Run time

50 seconds

### Other Information

The target frequency range is 0-0.5 Hz. No filtering is to be applied to the output time series.

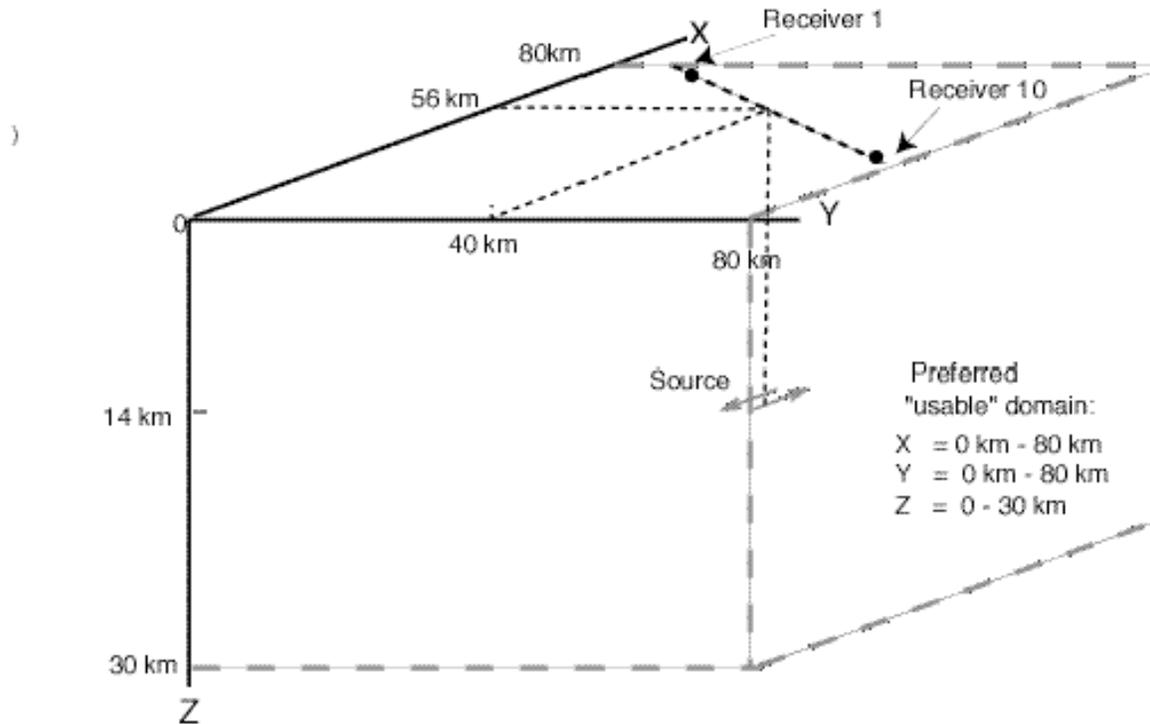
The “usable” part of grid (i.e., region where solution is considered relatively uncontaminated by artificial boundary effects) should be large enough to contain all receivers. Recommended usable volume is

$$0 < x < 80$$

$$0 < y < 80$$

$$0 < z < 30$$

Precise location of sponge zones and absorbing boundaries is not specified. However, added buffer zones and/or sponge zones should not add more than 25% in linear extent to the above recommended domain in any of the 3 coordinate directions.



**Fig. 1 Geometry for problem SC2.1**

## 2. Problem SC2.2

### Overview

This problem is for a complex 3D model, intended to be a realistic seismic velocity model for the San Fernando Valley/Los Angeles Basin region of southern California. The source-receiver geometry is sketched in Figure 2.

### Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

## Mapping to Geographical Coordinates

In order to map the southern California velocity structure (which is specified in lat, lon.) onto this coordinate system, map latitude onto meters north, and map longitude onto meters east, according to the following recipe:

SW corner is at (33.7275238 latitude, -118.90798187 long). This corner maps to origin, i.e.,  $(x,y) = (0\text{m}, 0\text{ m})$ .

NW corner is at (34.44875336, -118.90798187), and maps to  $(x,y) = (80000\text{ m}, 0\text{ m})$ .

SE corner, (33.7275238, -118.04201508), maps to  $(x,y) = (0\text{ m}, 80000\text{ m})$ .

NE corner, (34.44875336, -118.04201508), maps to  $(x,y) = (80000\text{ m}, 80000\text{ m})$ .

This mapping neglects earth curvature, and should be equivalent to  $1^\circ\text{ lat} = 110.922\text{ km}$ ,  $1^\circ\text{ lon} = 92.382\text{ km}$ .

Note: The above corners were specified because they were convenient for defining the coordinate mapping. Actual grid may be cropped within, or extended beyond, this 80 x 80 x 30 km region.

## Material Properties.

This problem uses the SCEC Southern California Seismic Velocity Model, Version 2, except for modifications described below to impose a lower limit on the velocities. The unmodified model is described in the following publication:

Magistrale, H., S. M. Day, R. Clayton, and R.W. Graves (2000). The SCEC southern California reference three-dimensional seismic velocity model version 2, *Bull. Seism. Soc. Am.*, 90, S65-S76.

The model is available as a Fortran code by anonymous ftp:

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The SCEC model is to be modified as follows for this computation: Replace the SCEC model S velocity with the value 500 m/s whenever the SCEC model value falls below 500 m/s. Whenever this minimum S velocity is imposed, the P wave velocity is set equal

to 3 times the S velocity (1500 m/s in this case). Density values follow the SCEC model without modification.

For this test,  $Q_p$  and  $Q_s$  will both be infinite.

### Source

[Source is similar to that of the previous point source problems, but with a different depth and different time constant ( $T$ ), corresponding to the increased spatial scale (and lower maximum frequency) of the calculation.]

Point dislocation. The only non-zero moment tensor component is  $M_{xy}$  (equal to  $M_{yx}$ ), which has value  $M_0=10^{18}$  Nm.

Moment-rate time history is  $M_0*(t/T^2)*\exp(-t/T)$ , where  $T=0.2$  sec.

(Equivalently, the moment time history is  $M_0*(1-(1+t/T)*\exp(-t/T))$ , where  $T=0.2$  sec).

Hypocentral coordinates (56000 m, 40000 m, 14000 m)

(As a check, this should put the epicenter slightly east of Northridge.)

### Receivers

Velocity time histories, in meters/sec, along free surface, at 8 points. The 8 points are along a line through the epicenter, oriented at angle  $-53.13$  (i.e.,  $-\tan^{-1}(4/3)$ ) to the x axis. The receiver coordinates are expressed here in km to make them easier to read (but all output files are still to be in mks units):

$(80 - j*6, 8 + j*8)$ ,  $j = 1, \dots, 8$

That is, the points

(74km, 16km)  
(68km, 24km)  
(62km, 32km)  
(56km, 40km)  
(50km, 48km)  
(44km, 56km)  
(38km, 64km)  
(32km, 72km)

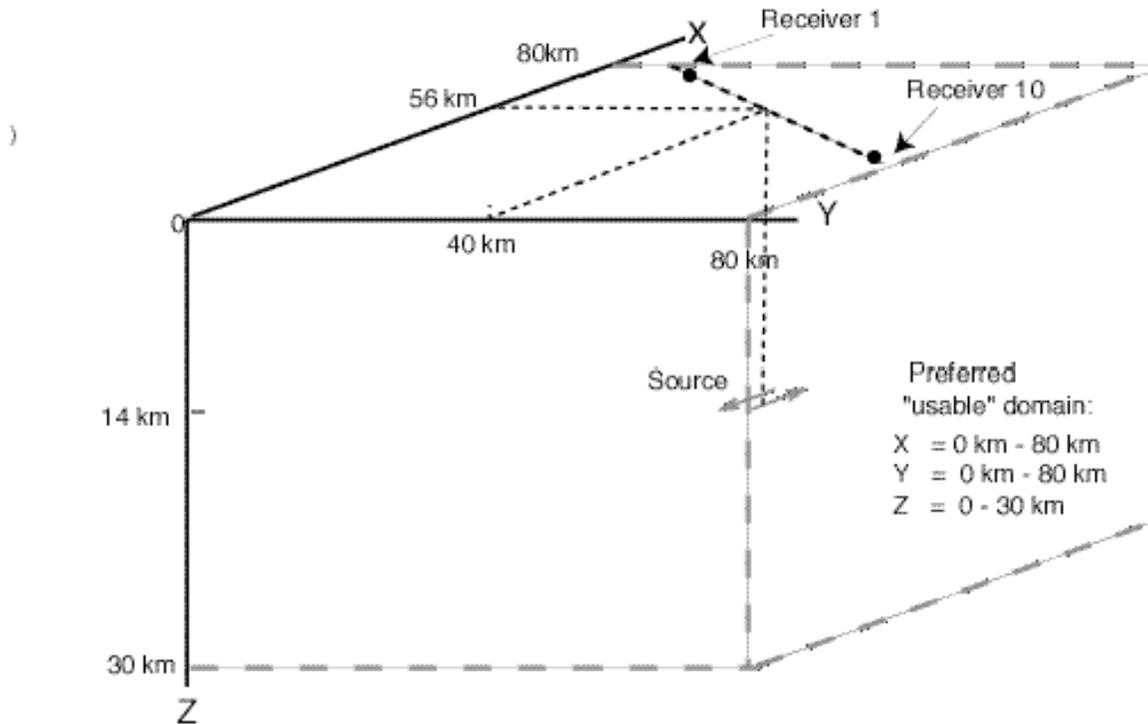
Velocity components are to be given in the same coordinate system as above, i.e.,  $v_x$  positive North,  $v_y$  positive East, and  $v_z$  positive down.

Output format is same as previously.

Output file suffix is SC2.2 (to indicate Southern California model, Version 2, problem number 2).

Run time

50 seconds



**Fig. 2 Geometry for problem SC2.2**

Other Information

The target frequency range is 0-0.5 Hz. No filtering is to be applied to the output time series.

The “usable” part of grid (i.e., region where solution is considered relatively uncontaminated by artificial boundary effects) should be large enough to contain all receivers. Recommended usable volume is

$$\begin{aligned} 0 < x < 80 \\ 0 < y < 80 \\ 0 < z < 30 \end{aligned}$$

Precise location of sponge zones and absorbing boundaries is not specified. However, added buffer zones and/or sponge zones should not add more than 25% in linear extent to the above recommended domain in any of the 3 coordinate directions.

### 3. Problem LOH.3

#### Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters. Problem geometry is shown in Figure 3.

#### Material Properties

The top 1000 m has  $V_s=2,000$  m/s,  $V_p=4,000$  m/s, density= $2600$  kg/m<sup>3</sup>.

The underlying halfspace has (as before)  $V_s=3,464$  m/s,  $V_p=6,000$  m/s, density= $2700$  kg/m<sup>3</sup>

$Q_s$  everywhere equal to  $0.02*(V_s \text{ in m/s})$   
 $Q_p$  everywhere equal to  $Q_s*(3/4)*(V_p/V_s)^2$   
(which implies  $Q(\text{bulk}) = \infty$ )

(The above formulation should give  $Q_s(\text{layer})=40$ ,  $Q_p(\text{layer})=120$ ,  
 $Q_s(\text{halfspace})=69.3$ ,  $Q_p(\text{halfspace})=155.9$ )

$Q(f)$  is frequency independent over the band 0.1 – 10 Hz. (Narrowband  $Q(f)$  approximations should aim for best fit in the 1 – 5 Hz band.)

The attenuation will introduce dispersion of the P and S wavespeeds; the target wavespeeds given above are for a reference frequency of 2.5 Hz.

#### Source:

Point dislocation. The only non-zero moment tensor component is  $M_{xy}$  (equal to  $M_{yx}$ ), which has value  $M_0=10^{18}$  Nm.

Moment-rate time history is  $M_0*(t/T^2)*\exp(-t/T)$ , where  $T=0.1$  sec.

(Equivalently, the moment time history is  $M_0*(1-(1+t/T)*\exp(-t/T))$ , where  $T=0.1$  sec).

Source Depth = 2000 m. That is, taking the epicenter as the origin, the source is at (0,0,2000).

#### Receivers:

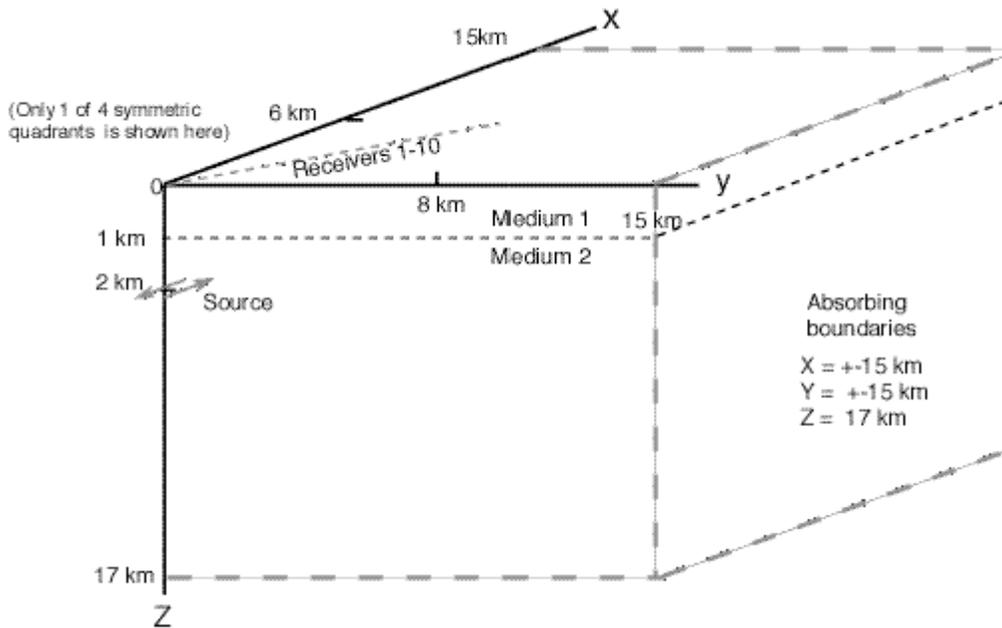
Velocity time histories, in meters/sec, along free surface, at the 10 points

(600,800,0)

(1200,1600,0)  
 (1800,2400,0)  
 .  
 .  
 etc, up to (6000,8000,0).

That is, receivers are at 1000 m intervals along line oriented at angle 53.13 degrees (i.e.,  $\tan^{-1}(4/3)$ ) to the x axis.

Velocity components are to be given in the same coordinate system as above, i.e.,  $v_x$  positive North,  $v_y$  positive East, and  $v_z$  positive down.



**Fig. 3 Geometry for problem LOH.3**

Run time

9 sec.

Other Information

*Mesh size.* Participants using uniform mesh 4/2 FD methods use a cell size of 100 m. for this test. Those using other methods should try to choose a cell size which will provide comparable accuracy.

*Artificial boundaries.* Place absorbing grid boundaries such that each boundary's orthogonal distance to the source point is 15,000 m. Distance of artificial boundaries of 15,000 m from source applies to all three directions, so, since the

source is 2000 m deep, the bottom boundary should be at 17,000 m depth. In the case of distributed absorbers, distance refers to distance to the nearest point at which some significant artificial reflection may be generated.

### Output Instructions

Solutions are to be compared with each other and with independent solutions over the bandwidth 0 to 5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function.

## **4. Problem LOH.4**

### Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

### Material Properties

The uppermost 1000 m has  $V_s=2,000$  m/s,  $V_p=4,000$  m/s, density= $2600$  kg/m<sup>3</sup>.

The underlying halfspace has  $V_s=3,464$  m/s,  $V_p=6,000$  m/s, density= $2700$  kg/m<sup>3</sup>

$Q_s$  and  $Q_p$  infinite

### Source

Finite Fault, with strike ( $\Delta$ )  $115^\circ$ , dip ( $\Delta$ )  $40^\circ$ , and rake ( $\Delta$ )  $70^\circ$  (see Fig 4a).

Angle and sign conventions follow Aki & Richards, p. 106.

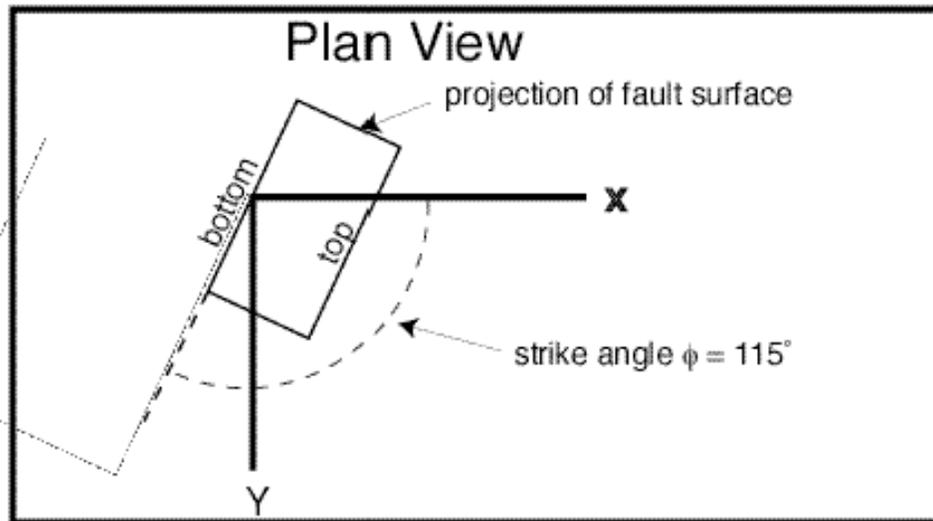
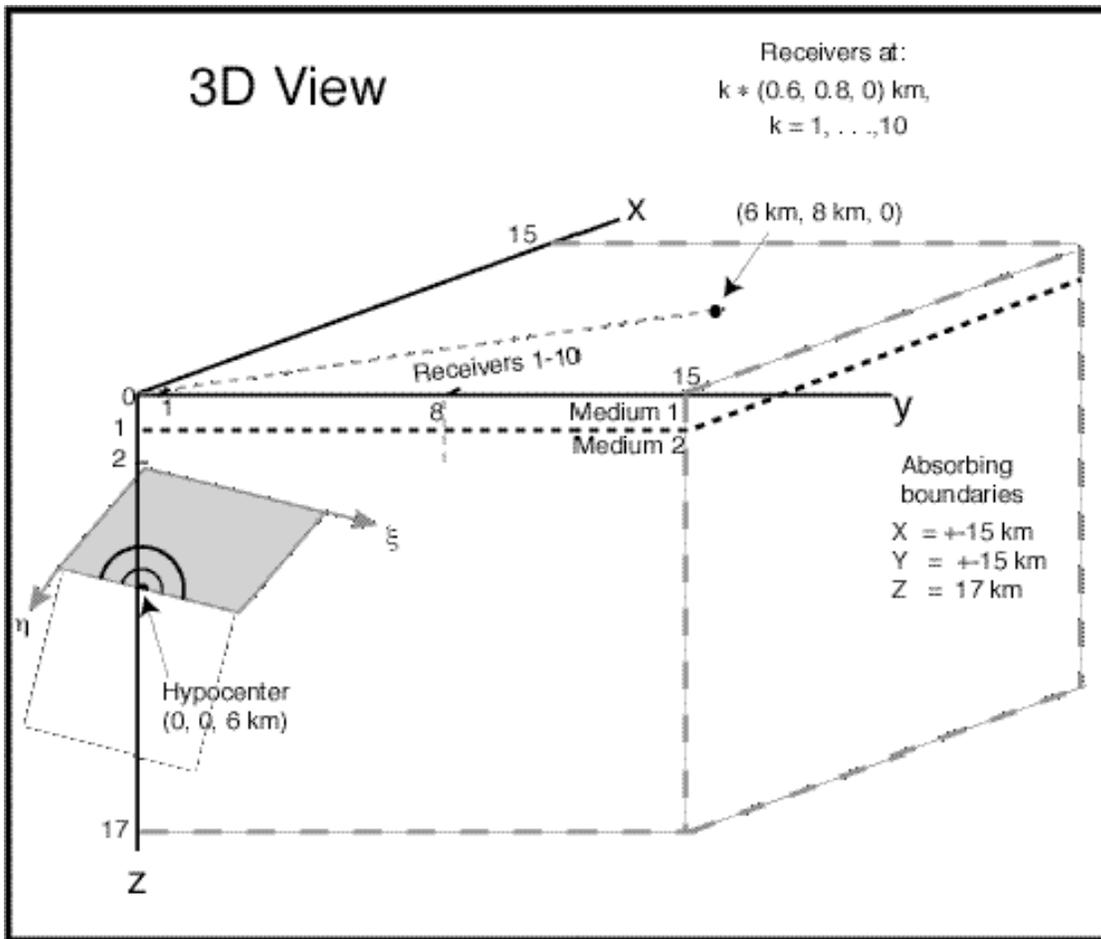
This rake angle corresponds to a thrust fault geometry (assuming positive slip).

Region of slip delimited by square 6000m on a side, with top and bottom aligned with strike direction. Center of the bottom-line of fault is located at (0,0,6000).

Hypocenter is centered along bottom of the fault, at  $(x_H, y_H, z_H) = (0, 0, 6000)$

Fault surface coordinates ( $\Delta$ ,  $\Delta$ ) aligned with strike direction and down-dip direction, respectively (see Fig 4b), with origin at NW corner.

Then, in the fault coordinate system, the hypocenter is denoted by  $(\Delta_H, \Delta_H)$ , and  $(\Delta_H, \Delta_H) = (3000, 6000)$ .



**Figure 4a. Global geometry for problem LOH.4**

In terms of fault-plane basis vectors  $\hat{u}, \hat{v}$ , the slip vector is  

$$\left[ \hat{u} \cos(\Delta) \hat{v} \sin(\Delta) \right] S(\Delta, t)$$
 (see Fig 4b), where the slip function  $S$  has same

shape and amplitude everywhere within the fault surface, but is time-shifted by an amount proportional to the distance of  $(\xi, \eta)$  from the hypocenter (i.e., the slip front propagates at a constant rupture velocity).  $S$  is given by

$$S(\xi, \eta, t) = S_0 \left[ 1 - \exp\left(-\frac{\tau}{T}\right) \right] H(\tau)$$

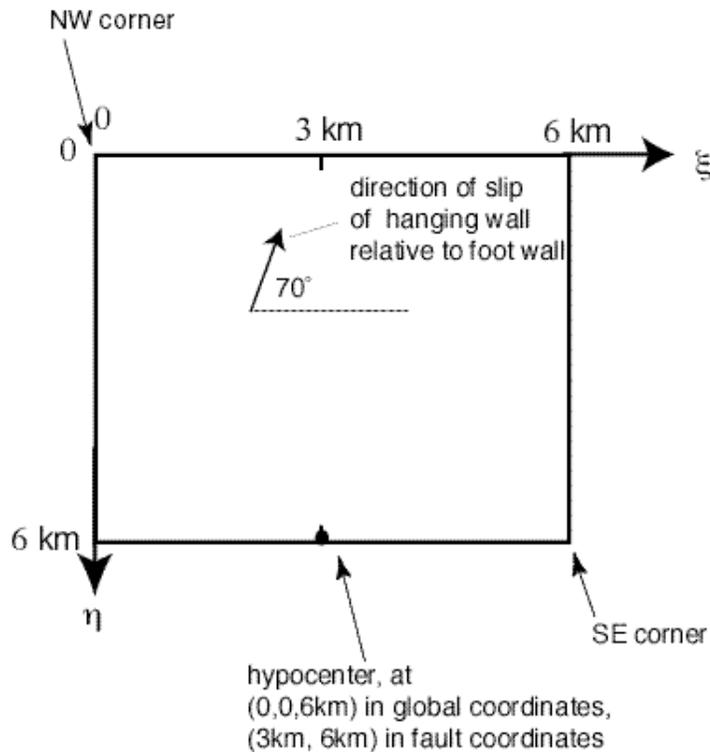
where  $H$  is the Heaviside step function, the time relative to rupture arrival,  $\tau$  is

$$\tau = t - \frac{r}{V_{rup}} = t - \frac{1}{V_{rup}} \left[ (\xi - \xi_H)^2 + (\eta - \eta_H)^2 \right]^{1/2},$$

the static slip  $S_0$  is 1 meter, the rupture velocity  $V_{rup}$  is 3000 m/sec, and the smoothing time,  $T$ , is 0.1 sec.

Equivalently, the slip velocity function is

$$\dot{S}(\xi, \eta, t) = S_0 \left( \frac{\tau}{T^2} \right) \exp\left(-\frac{\tau}{T}\right) H(\tau)$$



**Figure 4b. Fault-surface geometry for problem LOH.4**

Receivers:

Velocity time histories, in meters/sec, along free surface, at the 10 points

(600,800,0)

(1200,1600,0)

(1800,2400,0)

.

.

etc, up to (6000,8000,0).

That is, receivers are at 1000 m intervals along line oriented at angle 53.13 degrees (i.e.,  $\tan^{-1}(4/3)$ ) to the x axis.

Velocity components are to be given in the same coordinate system as above, i.e.,  $v_x$  positive North,  $v_y$  positive East, and  $v_z$  positive down.

### Run time

9 sec.

### Other Information

*Mesh size.* Participants using uniform mesh 4/2 FD methods use a cell size of 100 m. for this test. Those using other methods should try to choose a cell size which will provide comparable accuracy.

*Artificial boundaries.* Place absorbing grid boundaries such that each boundary's orthogonal distance to the source point is 15,000 m. Distance of artificial boundaries of 15,000 m from source applies to all three directions, so, since the source is 2000 m deep, the bottom boundary should be at 17,000 m depth. In the case of distributed absorbers, distance refers to distance to the nearest point at which some significant artificial reflection may be generated.

### Output Instructions

Solutions are to be compared with each other and with independent solutions over the bandwidth 0 to 5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function.

## 5. Problem NOR.1

### Coordinate System

Right-handed Cartesian, x positive north, y positive east, z positive down, all coordinates in meters.

### Mapping to Geographical Coordinates

In order to map the southern California velocity structure (which is specified in lat, lon.) onto this coordinate system, map latitude onto meters north, and map longitude onto meters east, according to the following recipe:

SW corner is at (33.7275238 latitude, -118.90798187 long). This corner maps to origin, i.e.,  $(x,y) = (0\text{m}, 0\text{ m})$ .

NW corner is at (34.44875336, -118.90798187), and maps to  $(x,y) = (80000\text{ m}, 0\text{ m})$ .

SE corner, (33.7275238, -118.04201508), maps to  $(x,y) = (0\text{ m}, 80000\text{ m})$ .

NE corner, (34.44875336, -118.04201508), maps to  $(x,y) = (80000\text{ m}, 80000\text{ m})$ .

This mapping neglects earth curvature, and should be equivalent to  $1^\circ\text{ lat} = 110.922\text{ km}$ ,  $1^\circ\text{ lon} = 92.382\text{ km}$ .

Note: The above corners were specified because they were convenient for defining the coordinate mapping. Actual grid may extend beyond, this 80 x 80 km region.

### Material Properties.

This problem uses the SCEC Southern California Seismic Velocity Model, Version 2, except for modifications described below to impose a lower limit on the velocities.

The unmodified model is described in the following publication:

Magistrale, H., S. M. Day, R. Clayton, and R.W. Graves (2000). The SCEC southern California reference three-dimensional seismic velocity model version 2, *Bull. Seism. Soc. Am.*, 90, S65-S76.

The model is available as a Fortran code by anonymous ftp:

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and its use is described in the file “read\_me” in the same directory. Questions concerning the SCEC velocity model should be directed to Harold Magistrale ([harold@moho.sdsu.edu](mailto:harold@moho.sdsu.edu)). Given a list of points, by lat (degrees), long (degrees), depth (meters), the code returns  $V_p$ ,  $V_s$  (both in m/s), and density ( $\text{kg/m}^3$ ). Details and examples are in the read\_me file.

The SCEC model is to be modified as follows for this computation: Replace the SCEC model S velocity with the value 500 m/s whenever the SCEC model value falls below 500 m/s. Whenever this minimum S velocity is imposed, the P wave velocity is set equal to 3 times the S velocity (1500 m/s in this case). Density values follow the SCEC model without modification.

For this test,  $Q_p$  and  $Q_s$  will be set as follows:

$$Q_s = 0.02 \times V_s \text{ (in m/s), when } V_s < 1500 \text{ m/s}$$

$$Q_s = 0.1 \times V_s \text{ (in m/s) when } V_s \geq 1500 \text{ m/s}$$

$$Q_p = 1.5 \times Q_s$$

### Source

Finite Fault, based on the Northridge earthquake model of Wald, Heaton, & Hudnut (BSSA, Vol 86, S49-S70, 1996), with some simplifications. Strike ( $\Delta$ )  $122^\circ$ , dip ( $\Delta$ )  $40^\circ$ . Rake ( $\Delta$ ) is  $101^\circ$  (see Fig 5)

Angle and sign conventions follow Aki & Richards, p. 106.

Region of slip delimited by rectangle 18000 m along strike, 24000 m along dip, with top and bottom aligned with strike direction. The (along-strike) center of the top edge of the fault is located at ( $34.344^\circ$  lat,  $-118.515^\circ$  lon, 5000 m depth).

Fault surface coordinates ( $\Delta$ ,  $\Delta$ ) aligned with strike direction and down-dip direction, respectively (see Fig 6, 7), with origin at NW corner.

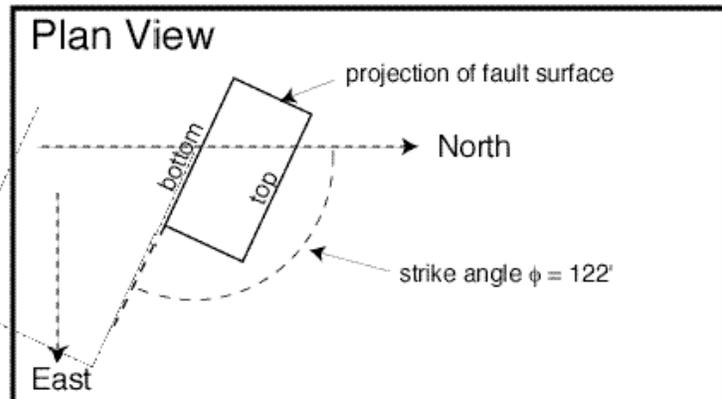
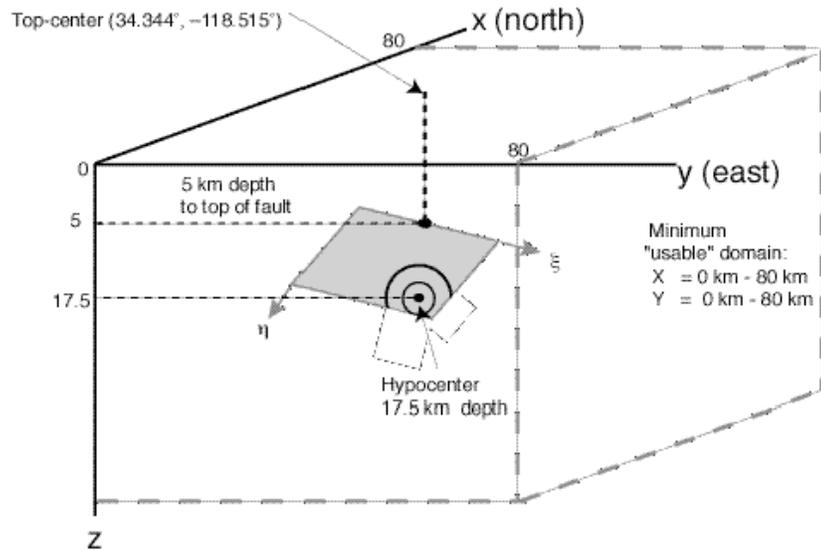
Then, in the fault coordinate system, the hypocenter is denoted by ( $\Delta_0$ ,  $\Delta_0$ ), and ( $\Delta_0$ ,  $\Delta_0$ ) = (15000 m, 19400 m).

### 3D View

Registration to SCEC Velocity Model:

(0,0) → (33.7275238 lat, -118.90798187 long)

(80 km, 80 km) → (34.44875336, -118.04201508)



**Figure 5. Global geometry for problem NOR.1**

In terms of fault-plane basis vectors  $\hat{u}, \hat{v}$ , and the (constant) rake angle  $\lambda$ , the slip-velocity vector is  $[\hat{u} \cos(\lambda) \hat{v} \sin(\lambda)] \dot{S}(x, y, t)$  (see Fig 6, 7). The slip velocity function  $\dot{S}$  has the same shape everywhere within the fault surface, but has an amplitude factor  $A(x, y)$  which varies with position, and a time shift proportional to the distance of  $(x, y)$  from the hypocenter (i.e., the slip front propagates at a constant rupture velocity). The slip velocity function,  $\dot{S}$ , is an isosceles triangle with duration  $T$  equal to 1 sec. The slip velocity function is thus given by

$$\dot{S}(\Delta, \tau, t) = A(\Delta, \tau) \left\{ (2\tau/T) [H(\tau) - H(\tau - T/2)] + (2 - 2\tau/T) [H(\tau - T/2) - H(\tau - T)] \right\}$$

where  $H$  is the Heaviside step function; the time relative to rupture arrival,  $\tau$  is

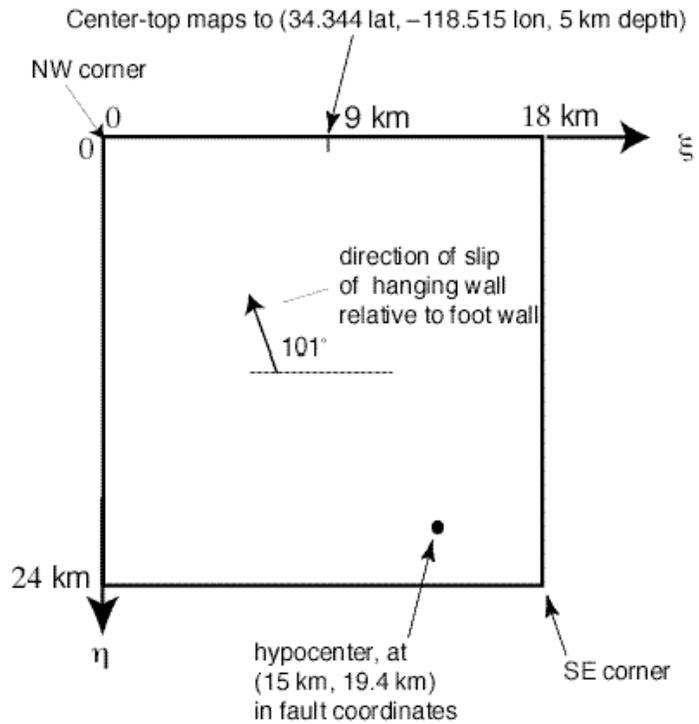
$$\tau = t - V_{rup} \left[ (\Delta - \Delta_H)^2 + (\tau - \tau_H)^2 \right]^{1/2},$$

and the rupture velocity  $V_{rup}$  is 3000 m/sec.

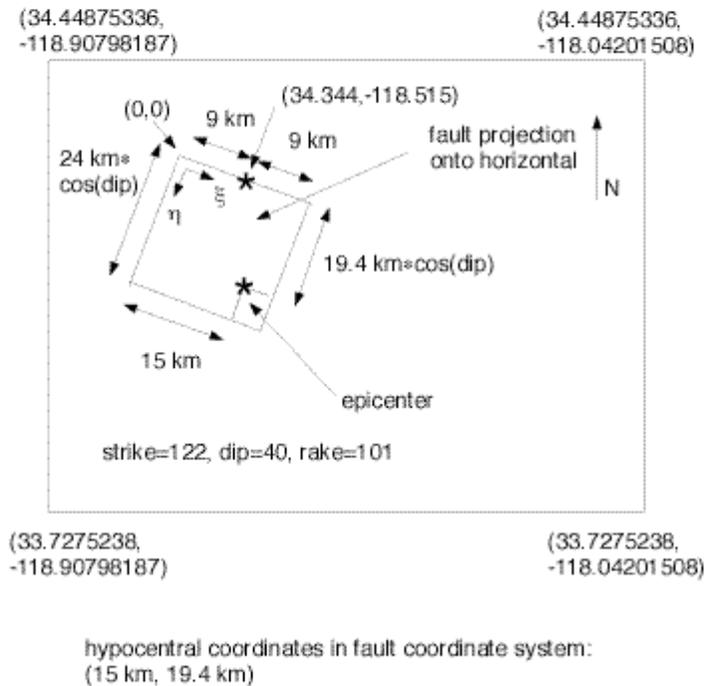
The amplitude function is piecewise constant on the fault plane, and is defined as follows. The fault surface is partitioned into  $\Delta_{\tau} \Delta_{\tau}$  identical, non-overlapping, rectangular elements, and  $A$  is constant over each of the rectangular elements. In our case, there are  $\Delta_{\tau} = 14$  columns of elements along strike, each with 18000/14 m along-strike dimension. There are  $\Delta_{\tau} = 14$  rows of elements along dip, each with 24000/14 m along-dip dimension. In each of the  $14^2$  sub-fault elements,  $A$  is scaled to give the same static slip  $S_{\infty}$  as the Wald et al. inversion (Wald et al., 1994, combined data-set solution), Table 1. That is, the values in Table 1, multiplied by  $2/T$ , give values of  $A$ . Note that the values in Table 1 are given in cm.

Table 1

NW (0, 0) (18 km, 0)		top										NE	
100.	34.	0.	0.	7.	13.	31.	55.	32.	32.	0.	0.	13.	7.
0.	55.	64.	24.	44.	59.	114.	51.	9.	69.	69.	36.	3.	31.
0.	107.	142.	162.	87.	89.	80.	33.	31.	127.	137.	90.	13.	73.
59.	128.	140.	163.	143.	117.	115.	89.	75.	91.	124.	92.	60.	87.
64.	89.	133.	232.	299.	252.	187.	160.	130.	84.	66.	60.	93.	129.
84.	79.	129.	221.	273.	231.	240.	238.	179.	118.	76.	50.	40.	105.
121.	129.	173.	248.	216.	185.	233.	264.	235.	189.	137.	100.	28.	44.
133.	132.	172.	216.	152.	130.	175.	237.	232.	186.	128.	51.	7.	80.
91.	101.	130.	161.	117.	83.	150.	185.	119.	147.	150.	100.	15.	0.
37.	95.	107.	91.	88.	32.	85.	107.	127.	209.	149.	123.	22.	0.
86.	120.	76.	59.	82.	30.	64.	75.	138.	155.	182.	268.	63.	0.
41.	55.	42.	44.	101.	51.	67.	103.	169.	58.	126.	177.	4.	0.
10.	36.	44.	57.	176.	127.	88.	70.	101.	45.	30.	31.	13.	53.
57.	76.	98.	58.	137.	114.	41.	31.	47.	40.	85.	113.	81.	52.
SW (0, 24km)		bottom										SE (18km, 24km)	



**Figure 6. Fault-surface geometry for problem NOR.1**



**Figure 7. Plan view of NOR.1. Coordinates  $\xi$  and  $\eta$  (shown projected onto horizontal) run along-strike and down-dip, respectively.**

### Receivers:

Output file has velocity time histories, in meters/sec, along free surface, at the 11 points shown in Table 2 (see Figure 8). Stations are in the order in which they are to be written to the file. Locations are given in geographical coordinates (the corresponding strong motion station id's are also shown), but should be written to the file in the problem coordinate system, i.e., in meters north and east.

Table 2

34.313000	-118.498001	JFP
34.230999	-118.712997	SSA
34.287998	-118.375000	PKC
34.090000	-118.338997	HSL
34.063000	-118.462997	VLA
33.904999	-118.278999	IGU
34.036999	-118.178001	OBG
33.919998	-118.137001	DWY
34.160000	-118.533997	TAR
34.087002	-118.693001	MCN
33.840000	-118.194000	LBL

Velocity components are to be given in the same coordinate system as above, i.e.,  $v_x$  positive North,  $v_y$  positive East, and  $v_z$  positive down.

### Run time

60 sec.

### Other Information

Solutions are to be compared with each other over the bandwidth 0 to 0.5 Hz. To insure uniformity in any comparisons, no additional filtering is to be applied to the time series apart from the specified source function. Output suffix is NOR.1.

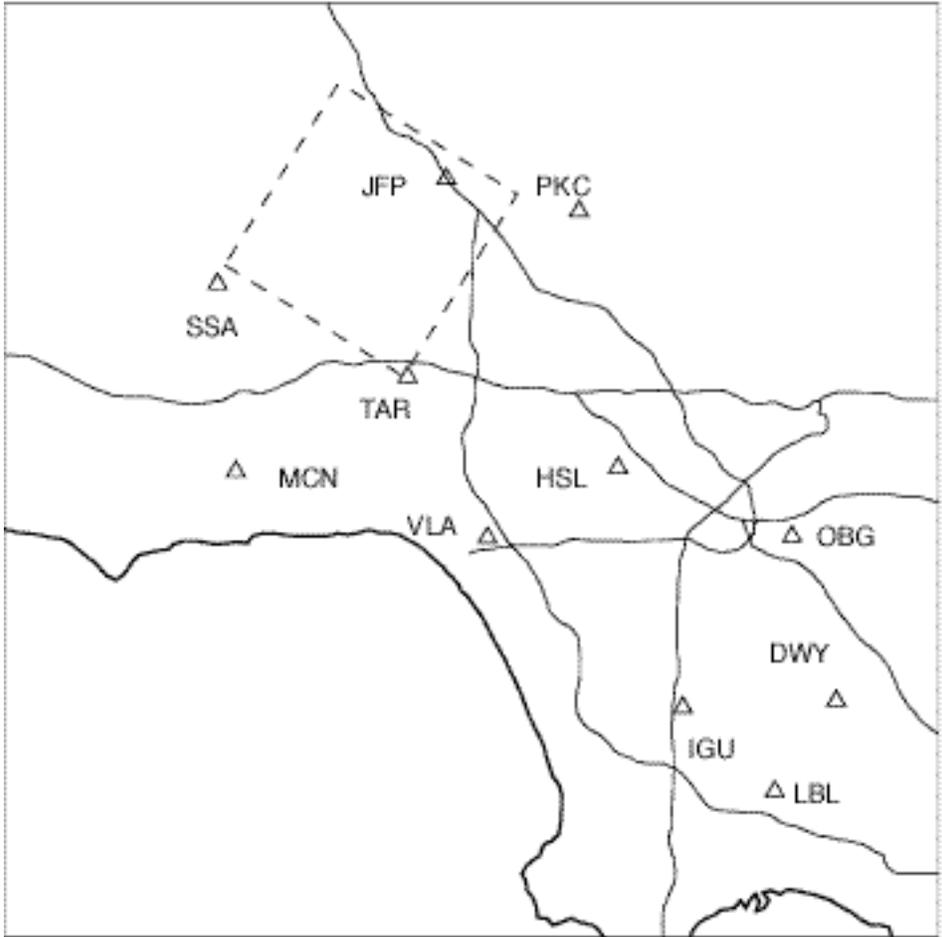


Figure 8. Station locations for NOR.1.

## OUTPUT FORMAT

Each problem generates one ascii output file. The file name consists of the 4-character abbreviation denoting the code used, followed by a dot and the problem name, e.g., “wcc2.uhs.1”.

The file format should be as illustrated by the write statements in the model Fortran code shown below. The Fortran model is followed by a Matlab m-file (macro) which reads the file produced by the Fortran program, and which can be used to test the file formatting. The file name 'PGETEST.1' in the line that reads `fid=fopen('PGETEST.1')` should be changed to the name of the file being read.

The output file will contain scalars `nr`, `nt`, and `dt`, which are the number of receiver points, number of time points in each time series, and time step size, respectively.

The output file also contains 2 1D arrays giving the output coordinate points. `x(i)` and `y(i)`,  $i=1:nr$ , contain the  $x$  and  $y$  coordinates of the receivers. For UHS.1, example,  $x(1)=600$ ,  $x(2)=1200$ , . . . ,  $x(10)=6000$ .

The rest of the output file consists of several 2D arrays giving the time series.

Array `t` contains the solution times, `t(j,i)` being the time associated with the solution at receiver  $i$  at time-point  $j$  (i.e., the  $nt*nr$  time values are written with the time index most rapidly varying). In the usual case of a constant time step, `t(j,i)` is just  $j*dt$ , for all  $i$ . Array `vx` contains  $x$ -component (positive North) velocities, `vx(j,i)` being the  $x$  velocity at time  $j$  at receiver point  $i$  (i.e., the time index is most rapidly varying).

Arrays `vy` and `vz` are the same as `vx`, but containing the  $y$  and  $z$  components of velocity, respectively.

### Fortran Illustration of Output

```
c*****  
dimension t(2000,100), vx(2000,100), vy(2000,100), vz(2000,100)  
dimension x(100),y(100)  
dt=.02  
dx=600.  
dy=800.  
pi=4.*atan(1.)  
period=1.0  
nr=10  
nt=1000  
do 1 i=1,nr  
x(i)=float(i)*dx  
y(i)=float(i)*dy  
amp=float(i)
```

```

do 1 j=1,nt
t(j,i)=float(j)*dt
vx(j,i)=amp*sin(pi*t(j,i)/period)
vy(j,i)=amp*cos(pi*t(j,i)/period)
vz(j,i)=amp*t(j,i)*exp(-t(j,i)/period)
1 continue
open (10,file='PGETEST.1')
rewind (10)
write(10,*) nr,nt,dt
write(10,*) (x(i),i=1,nr)
write(10,*) (y(i),i=1,nr)
write(10,*) ((t(j,i),j=1,nt),i=1,nr)
write(10,*) ((vx(j,i),j=1,nt),i=1,nr)
write(10,*) ((vy(j,i),j=1,nt),i=1,nr)
write(10,*) ((vz(j,i),j=1,nt),i=1,nr)
end

```

### Matlab macro to read file

```

% MATLAB macro
%
% Reads file PGETEST.1 generated by WriteFortran.f
%
% Test of ASCII format for PG&E/SCEC code verification
%
% Reads output generated by fortran program WriteFortran,
% which sits in /usr12/day/PGE/TestFormat
%
% S. Day 6jan99
%
fid=fopen('PGETEST.1')
A=fscanf(fid,'%g',[3]);
nr=A(1);nt=A(2);dt=A(3);
x=fscanf(fid,'%g',[nr]);
y=fscanf(fid,'%g',[nr]);
t=fscanf(fid,'%g',[nt,nr]);
vx=fscanf(fid,'%g',[nt,nr]);
vy=fscanf(fid,'%g',[nt,nr]);
vz=fscanf(fid,'%g',[nt,nr]);
fclose(fid);
for k=1:nr
figure(k)
plot(t(:,k),vx(:,k),'r',t(:,k),vy(:,k),'b',t(:,k),vz(:,k),'g')
end

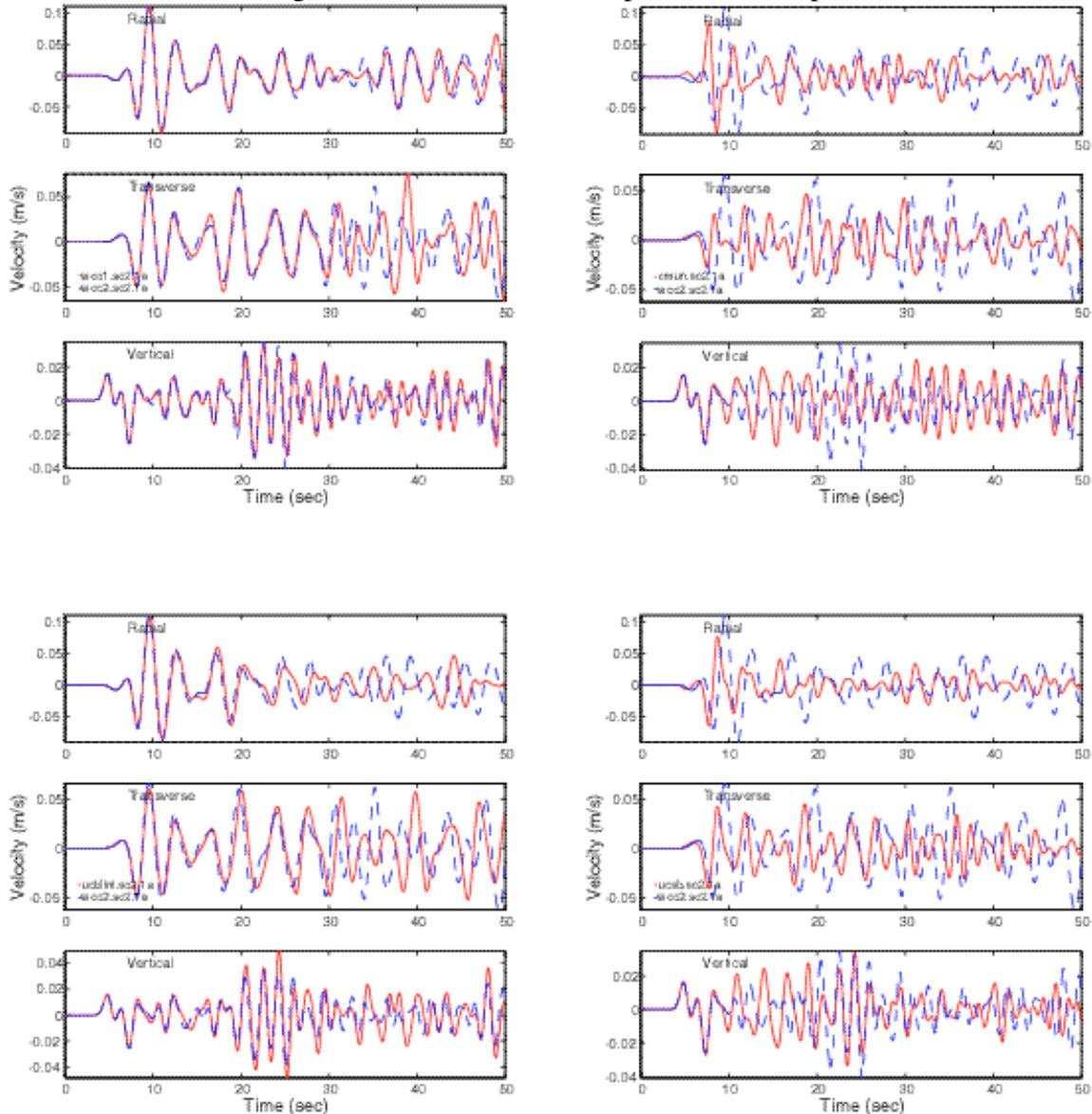
```

## TEST RESULTS

Representative solutions from SC2.1, SC2.2, LOH.3, and LOH.4 are shown in the following sections. Analysis of problem NOR.1 is being completed under follow-on Task 1A03.

### Problem SC2.1

Figure 9 shows representative results for Problem SC2.1, for the all 3 components of velocity at the receiver location (50km, 48km) (original source deconvolved and replaced with a Gaussian of spread 0.5 s). We have used the WCC2 solution as a common reference (blue curve) against which the others are plotted for comparison.



**Fig. 9. Comparison of solutions for SC2.1 at receiver location (50km, 48km), with the WCC2 solution (in blue) common to all plots, for reference.**

Substantial differences are evident among solutions, even in the initial arrivals. We find that the differences can be explained as a result of aliasing of the uppermost part of the SCEC seismic velocity structure when it is sampled at the 100 m mesh sampling interval used in the 4 FD codes. Recall that SC2.1 used the SCEC model, retaining seismic S velocities as low as 200 m/s. Velocities below about 500 m/s, when they are present at all, are mostly confined to the top 100 m or so of the model. As a result, large velocity gradients are present in the uppermost 100 m, and the 100 m meshing under samples this structure. The consequences are different for different FD codes, as a result of differences in way the field variable locations are staggered with respect to the free surface. It appears that this aliasing of the velocity model is a more severe restriction in the SCEC model than is undersampling of the wavefields themselves, in the 0-0.5 Hz frequency band considered here (i.e., in this band, the earth structure varies on a spatial scale short compared with the minimum seismic wavelength). This issue has received little or no consideration in the past, since earth models with the multi-scale resolution of the SCEC model have rarely been used in 3D modeling. The FE (CMUN) calculation, because its unstructured meshing permits a spatially variable sample interval, was able to sample the high gradient region at the top of the model with a  $\sim 30$  m sample interval. Therefore, the FE is likely the most accurate solution in this instance. We have been able to confirm this conjecture by comparing the FE solution with another FD code which permits the grid interval to vary in the vertical direction (thereby sampling the shallow seismic velocity gradients without aliasing).

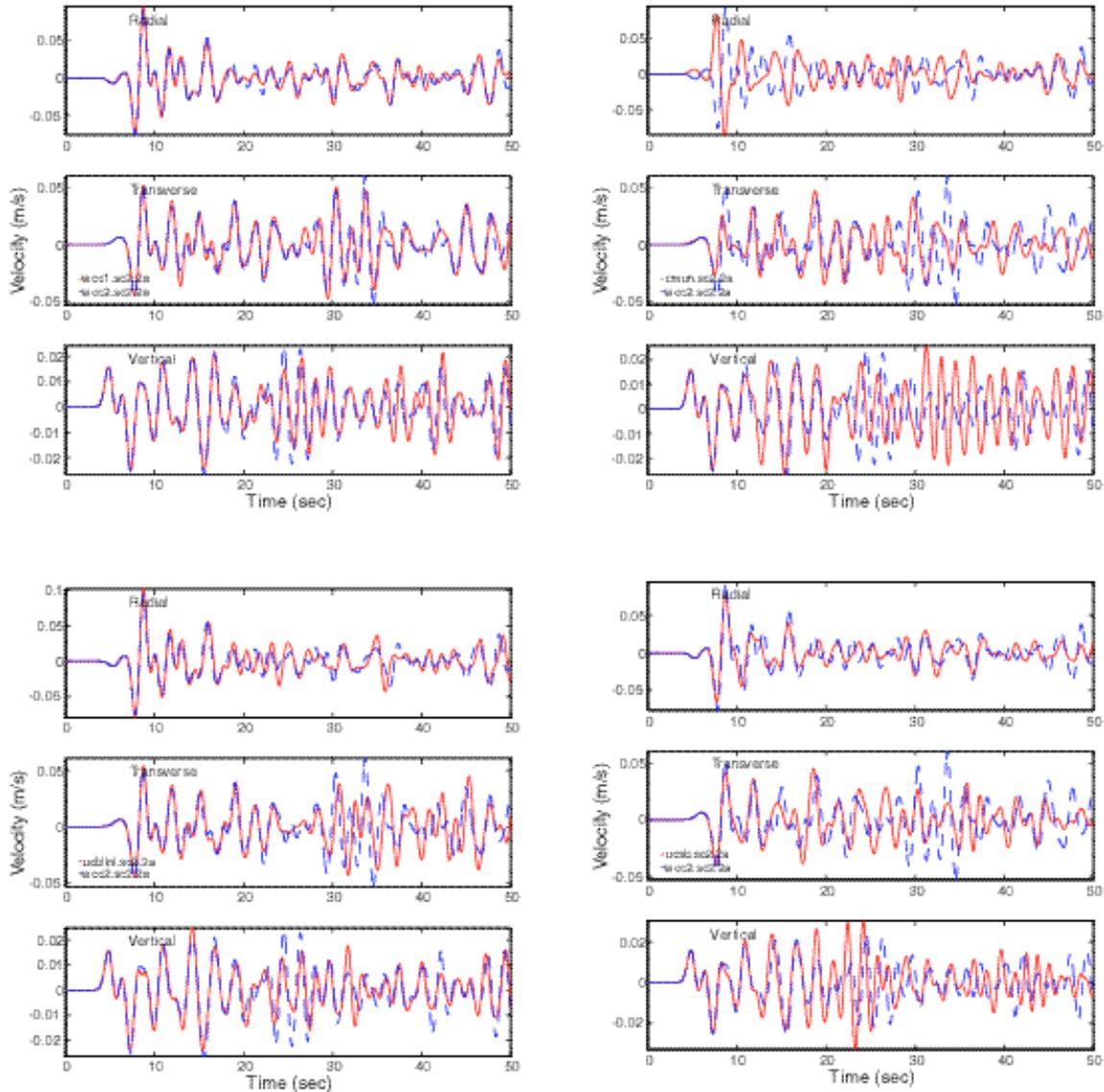
### **Problem SC2.2**

Figure 10 shows representative results for Problem SC2.2, for the all 3 components of velocity at the receiver location (50km, 48km) (original source deconvolved and replaced with a Gaussian of spread 0.5 s). This problem is identical to SC2.1, except that the minimum S wave velocity is 500 m/s instead of 200 m/s. We have used the WCC2 solution as a common reference (blue curve) against which the others are plotted for comparison.

In this case, there is good agreement among all methods for at least the first 20 seconds of record (at this distance from the source, which amounts to several tens of kilometers). It is our interpretation that this improvement in agreement results because eliminating the lowest seismic velocities in the model eliminates the high gradients. The result is a rather smooth shallow velocity structure that can be discretized at 100 m sample interval without aliasing. As a secondary effect, the elimination of the lowest seismic velocities also improves the sampling of the wavefield by the FD methods. However, the former effect dominates, as we have verified through examination of the solutions after further lowpass filtering.

Since (as noted above) we have evidence that the FE solution (CMUN code) is the most accurate solution in this particular case (because it most accurately represents the shallow part of the velocity model), we can assess whether including velocities lower than 500 m/s is critical to solution accuracy, in this particular model and this bandwidth ( $\sim 0-0.5$  Hz). This can be done by comparing the CMUN solutions to SC2.1 and SC2.2. In

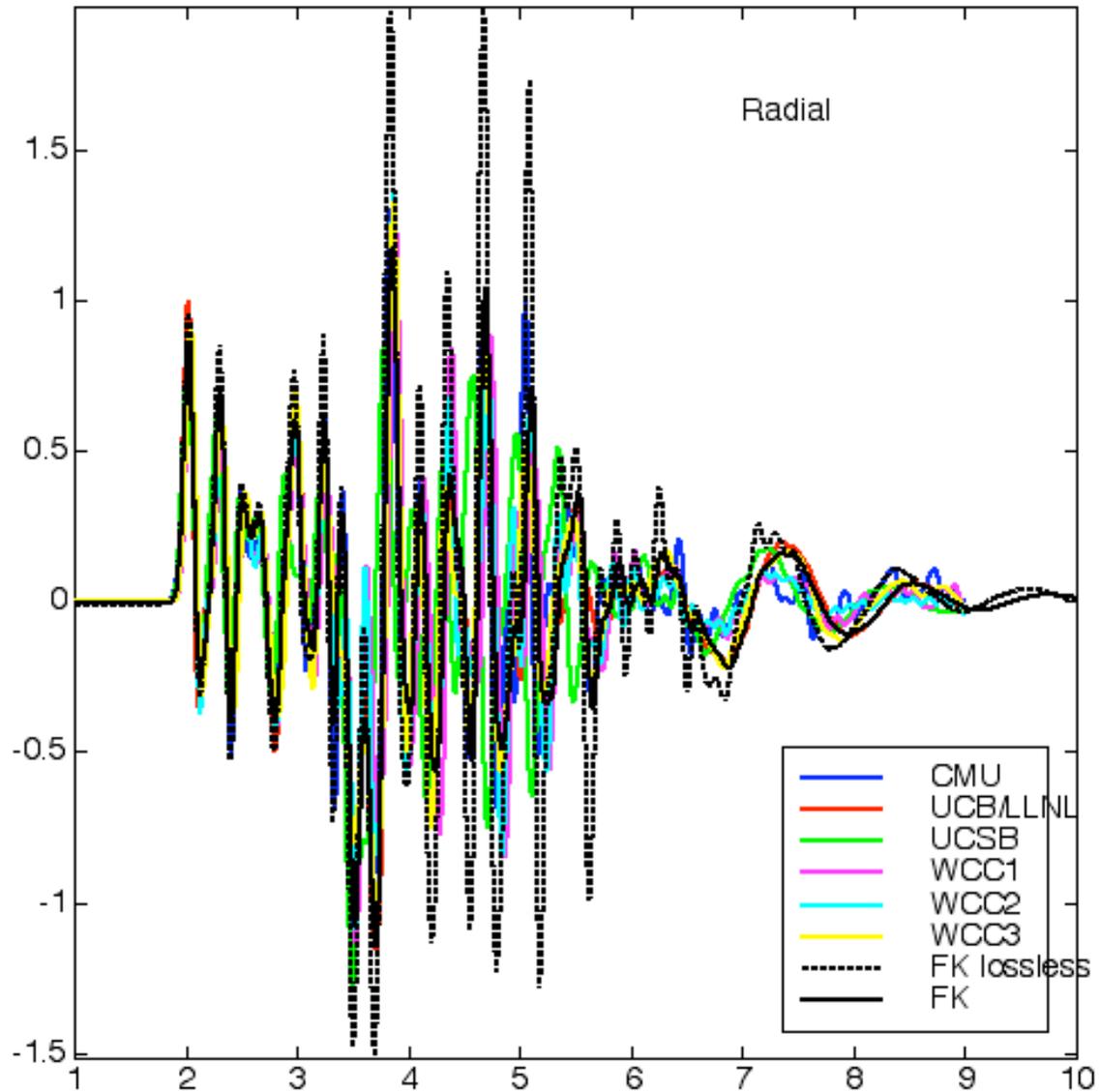
preliminary investigation of this question, we find differences which are relatively minor, rarely as much as 10 to 20 percent. We will pursue this question further for the final report, but the preliminary indication is that a 500 m/s velocity minimum can be imposed on the SCEC model without significantly compromising accuracy in the 0-0.5 Hz band.



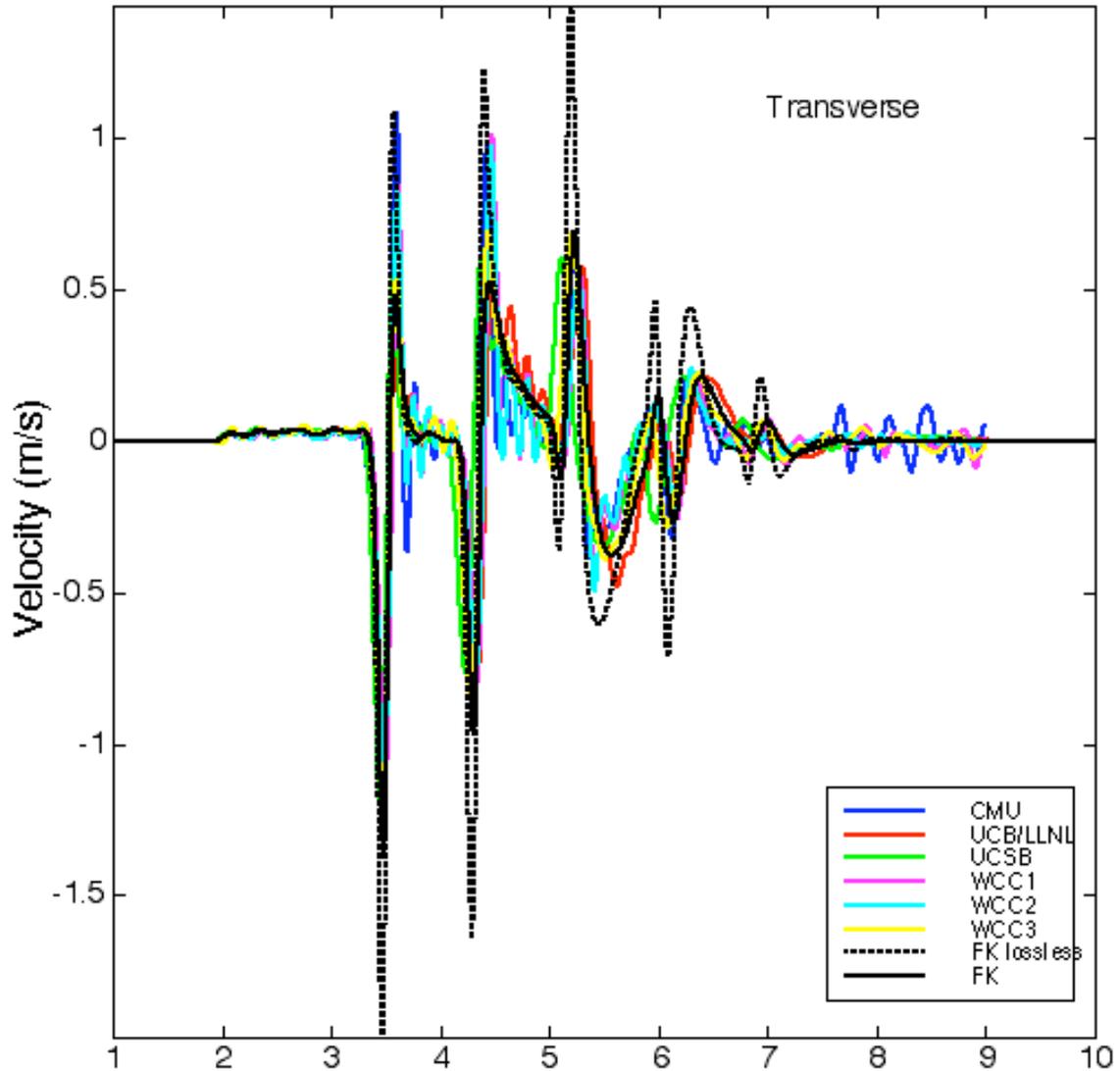
**Fig. 10. Comparison of solutions for SC2.2 at receiver location (50km, 48km), with the WCC2 solution (in blue) common to all plots, for reference.**

### Problem LOH.3

Figure 11 shows the results for the layer over halfspace test with anelastic attenuation, with point dislocation source (Gaussian time function, spread 0.05 s), Problem LOH.3. Parts a, b, and c, show the radial, transverse, and vertical components, respectively, at

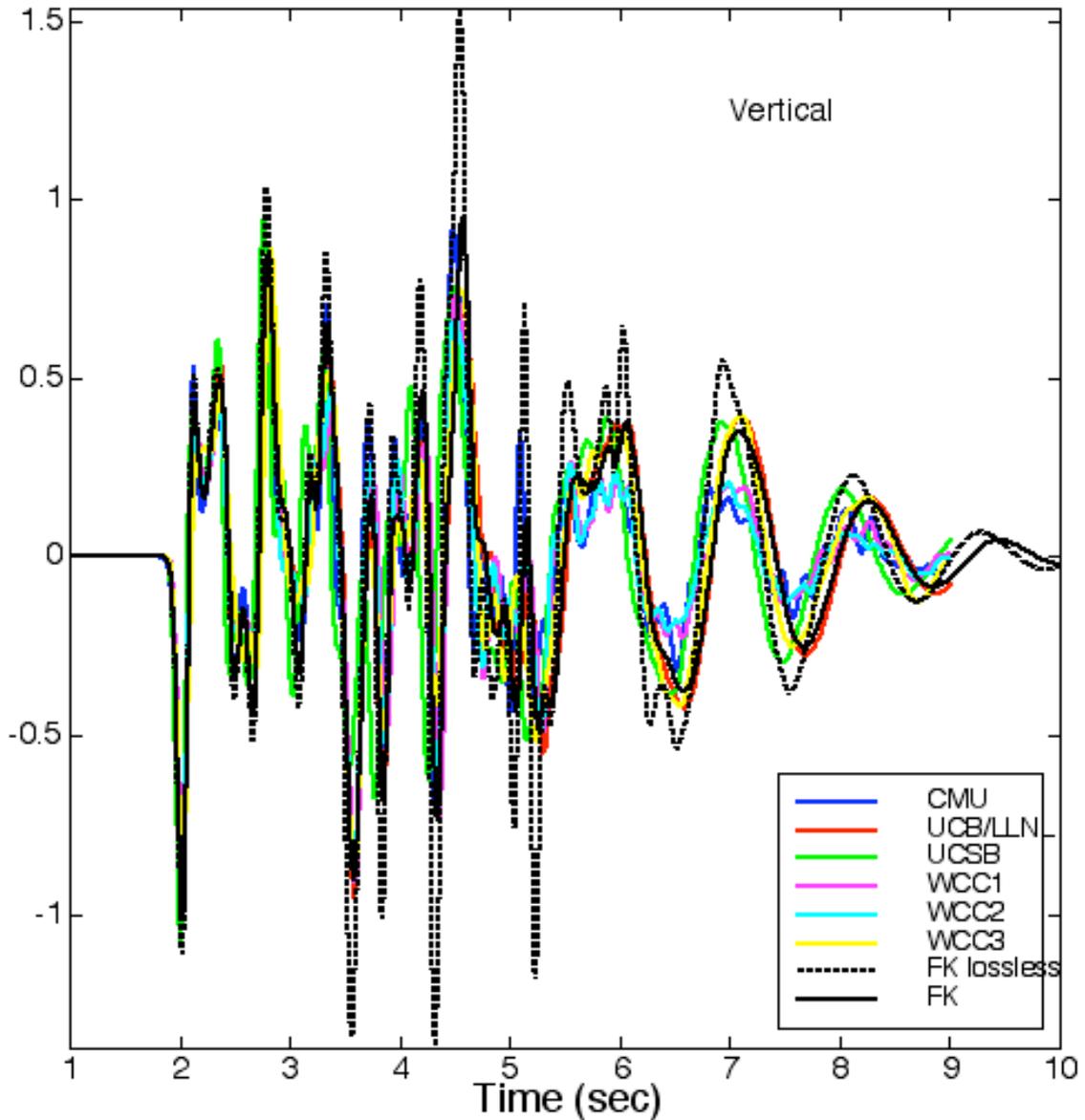


**Fig 11. (a) Radial components, at distance 10 km, for problem LOH.3. Shown in addition to solutions from the principal FE and FD codes are 3 other solutions. WCC3 is a version of WCC1, but with a newly implemented broadband attenuation model based upon the coarse-grain memory variables approach of Day and Bradley (2001). FK is the frequency-wavenumber solution using a modification of the method of Apsel and Luco (1979), and FK\_lossless is the frequency-wavenumber solution with infinite  $Q$ .**



**Fig 11. (b) Transverse components, at distance 10 km, for problem LOH.3.**

distance 10 km. The solid black curve shows the frequency-wavenumber (FK) solution, while the dashed black curve is the FK solution for infinite  $Q$ . The comparison of these two reference solutions shows that anelastic attenuation has an important effect on the wavefield. Comparison with the FD and FE solutions shows the following. (i) All solutions accurately model the anelastic losses at the center of the problem bandwidth (which controls the peak motions in this case). (ii) At low and high frequencies, some methods (UCBL, UCSB) match the reference solution well. (iii) The others (WCC1, CMUN, WCC2) overpredict (under-attenuate) the high frequencies and underpredict (over-attenuate) the low frequencies. This is the result of differences in the anelastic formulations used. The UCBL code uses a standard linear solid formulation tuned to the center of the band of interest and the UCSB code uses the coarse-grain method for representing a broad, constant  $Q$  absorption band. The other codes use methods equivalent to a Maxwell solid, in which  $Q$  is proportional to frequency, and they therefore can only approximate the target  $Q$  well at some intermediate frequency.



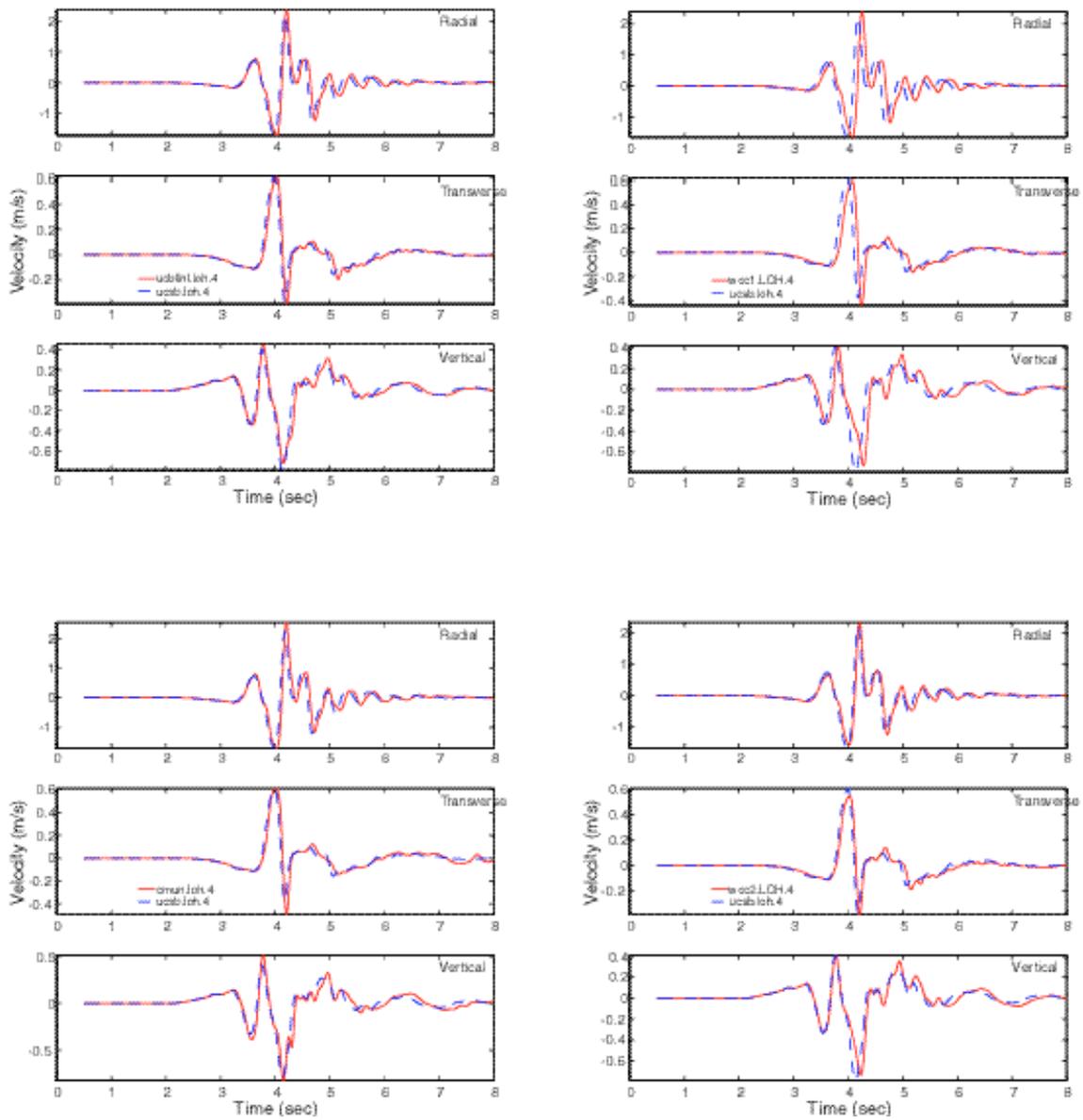
**Fig 11. (c) Vertical components, at distance 10 km, for problem LOH.3.**

Figure 11 also shows results labeled WCC3. This is the same FD code as WCC1, but with the attenuation model modified to incorporate the coarse-grain memory variables approach. As the figure shows, this modification renders the results comparable to those from UCSB and UCBL, as well as the reference FK solution.

#### **Problem LOH.4**

Figure 12 shows the results for the layer over halfspace test, with propagating thrust dislocation source (Gaussian time function of slip, spread 0.06 s). In this case, we have used the UCSB solution as a common reference (blue dashed curve) against which the others are plotted for comparison.

# PROPAGATING THRUST FAULT LAYER OVER HALFSPACE



**Fig 12. Velocity time histories, at distance 10 km, for problem LOH.4.**

## SUMMARY

Six new test problems have been designed and carried out under Task 1A02. Four have been fully analyzed and analysis of the final 2 (modeled on the 1994 Northridge earthquake) is in progress under Task 1A03. Together, these test simulations complement an additional 4 which were carried out as part of the initial phase (Task 1A01).

For perfectly elastic problems, agreement among the codes appears to be very good, for at least the first 20 seconds of record (at distances of several tens of kilometers), when two conditions are met. The first is the conventional one that the grid size has to be small enough to resolve the wavelength associated with the maximum frequency at which the comparison is made. The second, and sometimes more severe one for the SCEC velocity model (in the case of low frequency ground motion), is that the grid represent spatial variations in near-surface seismic velocities without significant aliasing. It may in practice be possible to circumvent the latter restriction by some form of smoothing (via, e.g., homogenization or effective media theory) of the model prior to discretization.

For the anelastic test, agreement among the codes appears to be very good over the domain in which the prescribed  $Q$  model is well represented by the respective codes. This is near the center of the prescribed absorption band. At the low and high frequency ends of the prescribed frequency band, those methods employing a broadband attenuation scheme agree closely among themselves and with the analytical solution, while significant departures from the analytical solution are evident for other methods.